**Second order linear homogeneous ODEs with constant coefficients**

Effects of increasing damping on a mass-spring system with constant restoring force

The response of the system represented by the initial value problem

$$\frac{d^2y}{dt^2} + 2a\frac{dy}{dt} + 100y = 0, \quad y(0)=0, \quad y'(0)=1$$

is

$$y(x) = \begin{cases} y = \dfrac{1}{\sqrt{100-a^2}} e^{-at} \sin\left(\sqrt{100-a^2}\, t\right) & (0 \le a < 10) \\ t e^{-10t} & (a = 10) \\ y = \dfrac{1}{\sqrt{a^2-100}} e^{-at} \sinh\left(\sqrt{a^2-100}\, t\right) & (a > 10) \end{cases}$$

Examples:

| | | | |
|---|---|---|---|
| Undamped: | $a=0:$ | $y = \frac{1}{10}\sin(10t)$ | (blue) |
| Underdamped: | $a=2:$ | $y = \frac{1}{\sqrt{96}} e^{-2t} \sin\left(\sqrt{96}\, t\right)$ | (purple) |
| | $a=6:$ | $y = \frac{1}{8} e^{-6t} \sin(8t)$ | (cyan) |
| | $a=8:$ | $y = \frac{1}{6} e^{-8t} \sin(6t)$ | (teal) |
| Critically damped: | $a=10:$ | $y = t e^{-10t}$ | (red) |
| Overdamped: | $a=12.5:$ | $y = \frac{1}{15}\left(e^{-5t} - e^{-20t}\right)$ | (black) |