1. Find the location (x, y, z) and nature (local minimum, local maximum or saddle point) of all critical points of the function [14]

$$f(x, y) = x^3 - 6xy + 3y^2$$

---

2 (a) Classify the conic section whose Cartesian equation is [2]

$$9x^2 + 25y^2 = 225$$

(b) Find the eccentricity and find the location (where they exist) of the foci and vertices of this conic section. [5]

(c) Sketch this conic section, showing the locations of the foci and vertices. [3]

(d) This conic section is rotated about the *x* axis. Write down the equation of the resulting surface of revolution. [3]

(e) The surface of revolution is also a quadric surface. Classify it. [3]

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3. Find the minimum value of the function $f(x, y) = x^2 + y^2$ subject to the constraint $g(x, y) = xy = 8$. [14]

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4. Find the interval of convergence for the power series [14]

$$f(x) = \sum_{n=1}^{\infty} \frac{(x-2)^n}{\sqrt{n}}$$

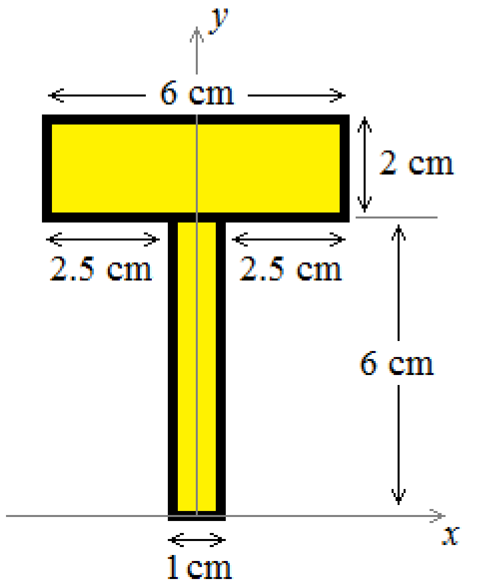
---

5. Find the Fourier sine series on the interval $0 < x < 2$ for the function [14]

$$f(x) = 2x - x^2$$

---

6. A T-bar of constant density has a cross section with the dimensions shown.



(a) Show that the centroid of the cross section is at $(\bar{x}, \bar{y}) = \left(0, \frac{17}{3}\right)$ (cm) [6]

(b) Find the centroidal second moment of area I_y for this T-bar cross section. [8]

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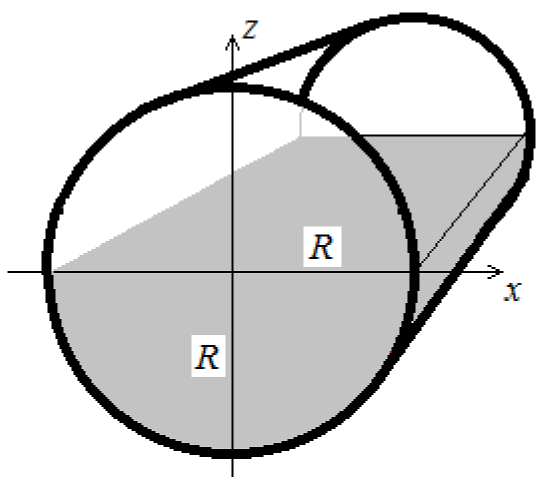
7. A tank in the shape of a right circular cylinder of cross sectional radius R is lying on its curved side and is filled up to the half-way point with incompressible fluid of density ρ. Find the hydrostatic force on the semi-circular end wall due to the fluid (as a function of ρ, g and R). Use a plane polar coordinate system whose pole (origin) is at the centre of the circle. [14]



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