ENGI 3425 Mathematics for Civil Engineering I

Practice Questions for the Final Examination

Many of these questions are based on questions in tests and final examinations from previous instructors of ENGI 3425: Dr. Ahmad Ghasemloonia (2013) and Dr. Weimin Huang (2010-12).

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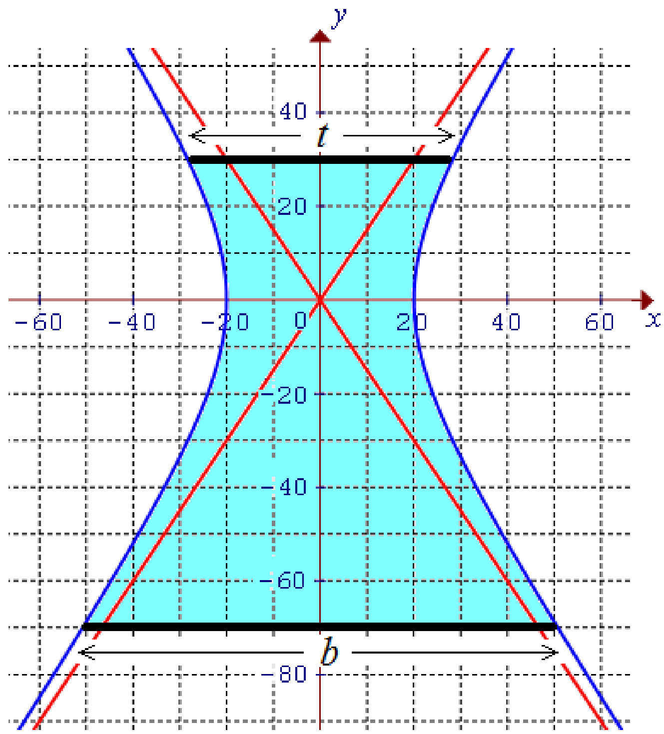
1. A nuclear cooling tower has the shape of a hyperboloid of one sheet; that is, the surface of revolution of a hyperbola around its conjugate axis, as shown below. The tower is 100 m tall, the top is 30 m above the centre of the hyperbola and the base is 70 m below the centre. The vertices of the hyperbola are located at $(\pm 20, 0)$ and the centre is at the origin. The asymptotes are $y = \pm 1.5x$.


Nuclear cooling tower



(a) Find the equation of the hyperbola in its standard form.
(b) What are the diameters of the top (t) and the base (b) of the tower?

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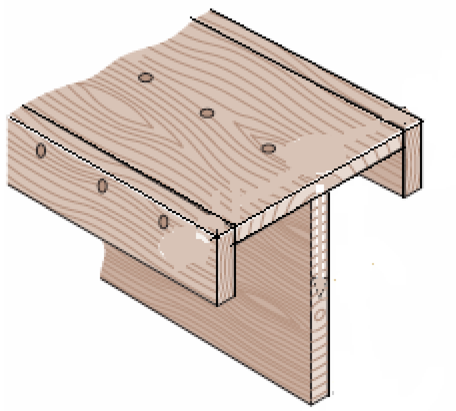
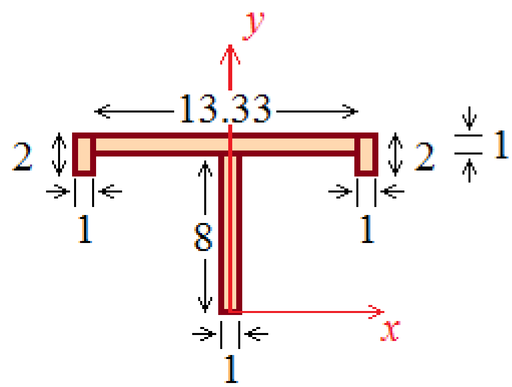
2. In order to determine the shear flow developed in each nail in the built-up member shown and in order to determine the nail spacing, it is required to determine the area moment of inertia $\left(I_y\right)$ of the entire cross-sectional area about the horizontal axis passing through the centroid. The material is homogenous (constant density throughout).





(a) Determine the location of the centroid of the cross-sectional area.

(b) Find the horizontal centroidal area moment of inertia $\left(I_x\right)$ of the cross-sectional area. The unit of length is cm (centimetres).

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3. A quadric surface is defined by

$$z = \frac{x^2 + y^2}{2}$$

(a) Classify this quadric surface.

(b) A solid V is bounded above by this quadric surface, below by the x-y plane and laterally by the cylinder of radius 2, centre the z-axis (equation $x^2 + y^2 = 4$). Use the cylindrical polar coordinate system to evaluate

$$I = \iiint_V \left(x^2 + y^2\right)^{3/2} dV$$

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4. Find the radius of convergence and the interval of convergence for the power series

$$f(x) = \sum_{n=1}^{\infty} \frac{(3-x)^n}{2^{2n}\sqrt{n}}$$

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5. For the quadric surface whose Cartesian equation is

$$x^2 - 4y^2 - 4z^2 = 4$$

(a) Classify the quadric surface and write down the locations of the vertices.

(b) Use differentials to find the approximate change in the value of z caused by moving on the surface from the point where $(x, y) = (3, 1)$ and $z > 0$ to the point where $(x, y) = (3.02, 1.01)$ and $z > 0$.

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6. Determine the location (x, y, z) and nature of all critical points of the function z, where

$$z = f(x, y) = x^2 - 6x + y^2 + 8y + 20$$

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7. Find the point on the oblate spheroid $x^2 + y^2 + 9z^2 = 1$ at which the sum of the x and y coordinates is the greatest.

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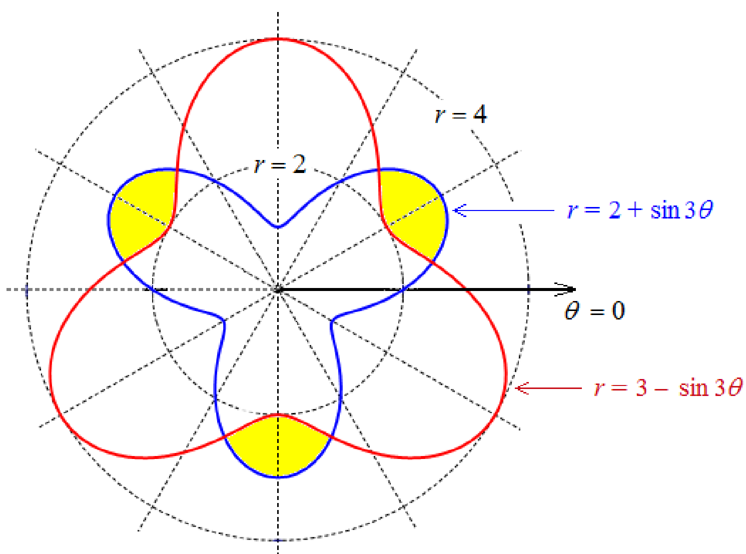
8. The region inside the curve $r = 2 + \sin 3\theta$ and outside $r = 3 - \sin 3\theta$ consists of three separate pieces. Find the area of one of these pieces.



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9. Is this series absolutely convergent, conditionally convergent or divergent?

$$\sum_{n=0}^{\infty} (-1)^n \frac{(n+2)!}{10^n \, n!}$$

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10. Find the exact value of the directional derivative of the function $f(x, y, z) = x^2 + xy^2 + z$ at the point $(1, 2, -1)$ in the direction of the vector $\vec{\mathbf{a}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$

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11. Evaluate the integral $I = \iint_R y\sqrt{x^2 + y^2}\, dA$, where R is defined as the region

$$R = \left\{(x, y) : 1 \le x^2 + y^2 \le 4 \text{ and } 0 \le y \le x\right\}$$

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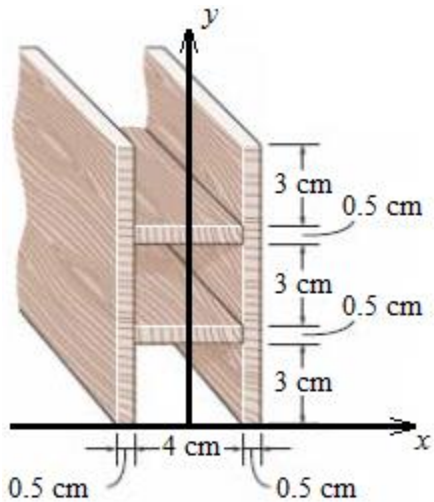
12. For the cross section of this shape,



(a) Find the location of the centroid, in terms of the coordinate axes shown.
(b) Find the centroidal area moments of inertia (I_x, I_y) of the cross-sectional area.

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13. The Cartesian equation of a conic section is

$$y^2 - 4y + 16 = 4x^2 + 8x$$

(a) Re-arrange this equation into standard form and hence classify this conic section.
(b) Find the eccentricity of this conic section and find the locations, where they exist, of the centre, vertices, foci, asymptotes and directrices.
(c) Sketch this curve.

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14. Find the interval of convergence for the power series $f(x) = \sum_{n=1}^{\infty} \frac{(x+2)^n}{n\,5^n}$

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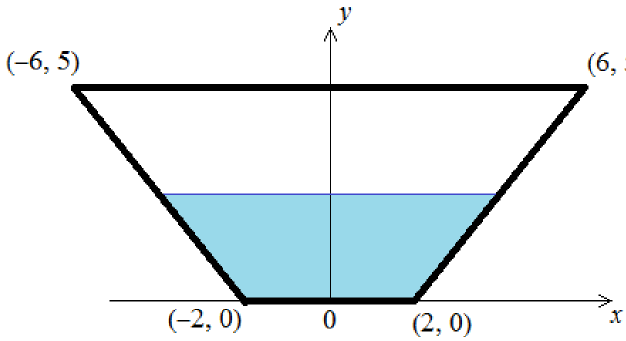
15. A tank has a trapezoidal cross-section as shown below and is filled with water to a height of 2.5 m. Find the fluid force on the trapezoidal surface due to water pressure. Use the values acceleration of gravity $g = 9.81\,\text{m/s}^2$ and water density $\rho = 1000$ kg/m^3.



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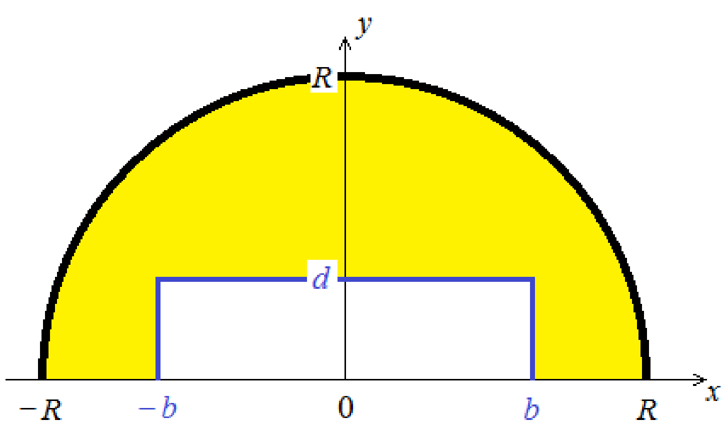
16. Find the area moment of inertia for the following section (semi-circle with rectangle cut out) about the *y*-axis. You may assume that $0 < b < R$, $0 < d < R$ and $b^2 + d^2 < R^2$.



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17. A rectangular box, open at the top, is to have a given capacity (i.e. volume) $V = 4\,\text{m}^3$. The material used to form the base and four sides of the box is of uniform thickness and density. Find the dimensions of the box requiring the least material for its construction.

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18. Find and simplify $I(x) = \int_e^x \frac{4}{t \ln(t^2)}\,dt$.

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19. Classify the conic section whose Cartesian equation is

$$2x^2 - 5xy + 3y^2 - x + y = 4$$

You may assume that the conic section is not degenerate.

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20. A conic section has the Cartesian equation

$$x^2 = 4y^2 + 16$$

(a) Classify this conic section and find its eccentricity.

(b) The conic section is rotated around the x-axis. Write down the equation of the resulting surface of revolution.

(c) The surface of revolution is also a quadric surface. Classify it.

(d) Find and simplify an integral expression for the curved surface area of revolution of this quadric surface between the plane $x = 5$ and the nearer vertex. Do ***not*** evaluate this integral.

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