ENGI 3425 Mathematics for Civil Engineering 1

Problem Set 10 Questions

(Chapter 9 – Introduction to Ordinary Differential Equations)

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1. Which of the following ordinary differential equations are linear?

(a) $x^2 \frac{d^4 y}{dx^4} - 3\frac{d^3 y}{dx^3} + 4x\frac{d^2 y}{dx^2} - \frac{dy}{dx} + e^x y = e^{-3x} \sin 4x$

(b) $(y-3)\frac{dy}{dx} = x\left(3y - y^2\right)$

(c) $2\frac{d^2 y}{dx^2} = \left(\frac{dy}{dx}\right)^2$

(d) $\frac{d^2 y}{dx^2} = \frac{x}{y}$

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2. The location $x(t)$ of an object moving under the influence of a restoring force and friction only (such as a spring) is modelled by the initial value problem

$$\frac{d^2 x}{dt^2} + 2\frac{dx}{dt} + 26x = 0$$

with the additional information that the object is released from rest at the location $x = 10$.
Verify that $x(t) = 2e^{-t}\left(5\cos 5t + \sin 5t\right)$ is the solution to this initial value problem.

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3. Verify that $y(x) = \cos(2x) - 2\cosh x$ is a solution to the fourth-order linear ordinary differential equation

$$\frac{d^4 y}{dx^4} + 3\frac{d^2 y}{dx^2} - 4y = 0$$

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4. Verify the following feature of a linear homogeneous [right side = 0] second order ordinary differential equation: Given that $y(x) = u(x)$ and $y(x) = v(x)$ are both solutions of

$$\frac{d^2 y}{dx^2} + p(x)\frac{dy}{dx} + q(x)\,y = 0$$

show that $y(x) = Au(x) + Bv(x)$ is also a solution for *any* values of the constants *A*, *B*.

This key feature distinguishes linear homogeneous ODEs from non-linear ODEs.
Any linear combination of solutions to a linear homogeneous ODE is also a solution of that ODE.

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