Conic Sections and Quadric Surfaces

1.   Classify the following conic sections, identify the major axis, centre, foci, vertices, directrices and asymptotes (where applicable) and sketch the conic section.

(a) $16x^2 - 9y^2 = 144$

(b) $16x^2 + 9y^2 = 144$

(c) $16x - 9y^2 = 0$

(d) $16x^2 - 9y^2 = 0$

(e) $9y^2 = 4x^2 + 8x + 36y - 32$

(f) $2x^2 + y^2 + 4x + 4y + 7 = 0$

---

2.   Find the Cartesian equation of the ellipse of eccentricity $e = 0.3$ whose directrices are the lines $x = -8$ and $x = 12$ and whose centre lies on the x axis. Sketch the ellipse, showing the locations of the foci, centre, vertices and directrices.

---

3.   A bridge above a road is supported by an arch in the shape of the upper half of an ellipse. The peak of the arch is 3 m above the centreline of the road. The span of the arch from the vertical wall by one side of the road to the vertical wall by the other side is 10 m. The road requires enough headroom to allow vehicles of height 2.5 m to pass under the arch. How wide (to the nearest 0.1 m) can the paved portion of the road be?

Also find the eccentricity e of this ellipse.

---

4.   Classify the following conic sections.

(a) $x^2 + 2xy + 3y^2 = 4$

(b) $x^2 - xy + y^2 + 1 = 0$

(c) $xy + y^2 = 0$

(d) $xy + y^2 - 4 = 0$

(e) $xy + y^2 + 4 = 0$

(f) $4x^2 - 4xy + y^2 + x = 1$

☞

5.   Classify and sketch the following quadric surfaces
   (a) $x^2 - 2y^2 + 3z^2 = 4$
   (b) $x^2 - 2y^2 + 3z^2 = -4$
   (c) $x^2 - 2y^2 + 3z^2 = 0$
   (d) $x^2 + 2y^2 + 3z^2 = -4$
   (f) $x^2 + 2y^2 + 3z^2 = 4$
   (g) $16x^2 + 9y^2 = 144$
   (h) $16z - 9y^2 = 0$
   (i) $16z + 9x^2 - 9y^2 = 0$