Arc Length, Surfaces of Revolution, Area

1. The position $\vec{\mathbf{r}}(t)$ (m) at any time t (s) of a particle travelling along a curve C is

$$\vec{\mathbf{r}}(t) = 6t\,\hat{\mathbf{i}} + 5t^2\hat{\mathbf{j}} - 8t\,\hat{\mathbf{k}}$$

(a) Find the velocity $\vec{\mathbf{v}}(t)$ and the speed $v(t)$. Hence find the unit tangent vector $\hat{\mathbf{T}}(t)$.

(b) Find the distance that the particle travels along the curve between the origin and the point $P(12, 20, -16)$. You may quote the standard integral

$$\int \sqrt{a^2 + x^2}\,dx = \frac{1}{2}\left(x\sqrt{a^2 + x^2} + a^2 \ln\left(x + \sqrt{a^2 + x^2}\right)\right) + C$$

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2. Find the length of the arc of the curve $r = \theta^2$ from the pole to the point $(r, \theta) = \left(\left(\frac{\pi}{2}\right)^2, \frac{\pi}{2}\right)$ **and** find the area swept out by this polar curve between these two points. Assume SI units.

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3. Find the arc length along the curve defined parametrically by $\vec{\mathbf{r}} = \left(t^3 + 2\right)\hat{\mathbf{i}} + \left(3t^2 + 1\right)\hat{\mathbf{j}}$ from the point where $t = 0$ to the point where $t = 2\sqrt{3}$. Assume SI units.

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4. For the curve C in the *xy* plane whose Cartesian equation is

$$y = 3\sqrt{1 - \frac{x^2}{16}}$$

(a) Classify and sketch the curve.

(b) Write down the equation of the surface of revolution formed by rotating this curve once around the *x* axis.

(c) Classify this surface of revolution; what type of quadric surface is it?

(d) Find and simplify (but do ***not*** evaluate) an integral expression for the total surface area of this surface of revolution.

(e) Find the volume enclosed by this surface of revolution.

(f) Use the parameterization $x = 4\cos t$, $y = 3\sin t$, $0 \le t \le \pi$ to find the area between the curve $y = f(x) = 3\sqrt{1 - \frac{x^2}{16}}$ and the *x* axis.

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5. By rotating the circle $x^2 + y^2 = a^2$ about the *x* axis, verify the equation of a sphere of radius *a* and the formulae for its surface area and for the volume enclosed by it.

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