ENGI 3425 Mathematics for Civil Engineering I

Problem Set 4 Questions

(Sections 6.01 – 6.06 – Sequences, Series, Tests for Convergence)

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1. Write down the next two terms and the general term a_n of the sequence

$$\{ a_n \} = \left\{ \frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \ldots \right\}$$

Determine whether or not the sequence converges and find the limiting value if it exists.

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2. Write down the next term and the general term b_n of the sequence

$$\{ b_n \} = \{ 84, 30, 12, 6, 4, \ldots \}$$

Determine whether or not the sequence converges and find the limiting value if it exists.

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3. Determine whether or not the sequence converges and find the limiting value if it exists.

$$\{ c_n \} = \left\{ \frac{n^3}{3^n} \right\}$$

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4. Write down the general term d_n of the sequence

$$\{ d_n \} = \left\{ +1, +2, -\frac{1}{3}, +\frac{3}{2}, -\frac{3}{5}, +\frac{4}{3}, -\frac{5}{7}, +\frac{5}{4}, -\frac{7}{9}, \ldots \right\}$$

Determine whether or not the sequence converges and find the limiting value if it exists.

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5. Prove that the divergence test is valid; that is, prove that the statement

$\lim_{n \to \infty} a_n \neq 0 \quad \Rightarrow \quad \sum_{n=1}^{\infty} a_n$ diverges (the sum does not exist)

is true.

Hint: examine the limit of $\left(S_n - S_{n-1} \right)$.

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6. Find the exact value of

$$s = \sum_{n=2}^{\infty} \frac{4}{n^2 - 1}$$

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7. Is this series absolutely convergent, conditionally convergent or divergent?

$$\sum_{n=1}^{\infty} (3n - 2)$$

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8. Is this series absolutely convergent, conditionally convergent or divergent?

$$\sum_{n=1}^{\infty} \frac{(-3)^n + (-2)^{2n}}{6^n}$$

If it converges, then evaluate its sum exactly.

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9. Is this series absolutely convergent, conditionally convergent or divergent?

$$\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{\sqrt[3]{1+n^2}}$$

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10. Write the recurring decimal $1.\dot{0}7\dot{0} = 1.070070070\ldots$ as a ratio of relatively prime integers.
[Hint: write the number as the sum of an infinite series]

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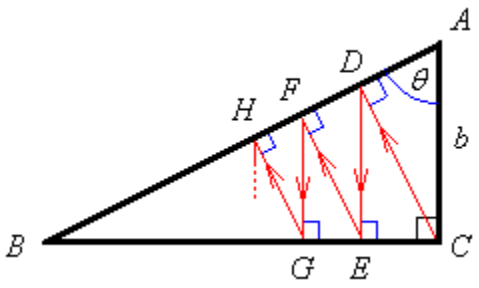
11. A particle bounces between sides BC and BA of the triangle ABC in such a way that it always travels the shortest distance possible to the next side (that is, at right angles to the next side), as illustrated below. The length of side AC is b, the angle at C is a right angle and the angle at A is θ. The particle sets out from C and arrives at B after infinitely many bounces. Find the total length L of the path of the particle in terms of b and θ.



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12. Is this series absolutely convergent, conditionally convergent or divergent?

$$\sum_{n=0}^{\infty} \frac{(-1)^n e}{e^n + 1}$$

Find an upper bound on the error in estimating the sum by the partial sum S_2.

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13. Is this series absolutely convergent, conditionally convergent or divergent?

$$\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$$

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14. Is this series absolutely convergent, conditionally convergent or divergent?

$$\sum_{n=0}^{\infty} (-1)^n \frac{(n+2)!}{n!10^n}$$

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15. Show that the ratio test fails for all *p*-series.

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