ENGI 3425 Mathematics for Civil Engineering I

Problem Set 5 Questions

(Sections 6.07 – 6.10 – Power Series, Binomial Series & Fourier Series)

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1. Find the radius R and interval I of convergence for the power series

$$f(x) = \sum_{n=1}^{\infty} \frac{(2x-6)^n}{\sqrt{n}}$$

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2. Find the radius R and interval I of convergence for the power series

$$f(x) = \sum_{n=0}^{\infty} \frac{n(x-1)^n}{2^n}$$

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3. Integrate the geometric series $\sum_{n=0}^{\infty} x^n$ and its sum with respect to x in order to find the Maclaurin series for $f(x) = \ln(1-x)$ and its radius of convergence.

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4. Find the binomial expansion of

$$f(x) = \sqrt[3]{1+x^3}$$

as far as the term in x^{11} and find its radius of convergence R.

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5. Find the binomial expansion of

$$f(x) = \frac{2x}{\sqrt{4-9x^2}}$$

as far as the term in x^7 and find its radius of convergence R.

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6. Find the interval of convergence I for the series

$$f(x) = \sum_{n=1}^{\infty} \frac{1}{n} \cdot \left(\frac{4}{x-3} \right)^{n/3}$$

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7. Find the power series expansion for $\dfrac{x}{(1-x)^2}$

Hence evaluate $S = \displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n}$ exactly.

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8. Use the binomial series expansion for $(1-x)^{-3}$ to show that the Maclaurin series for $f(x) = \dfrac{x+x^2}{(1-x)^3}$ is $\displaystyle\sum_{n=1}^{\infty} n^2 x^n$ and find its interval of convergence.

Hence evaluate $S = \displaystyle\sum_{n=1}^{\infty} \frac{n^2}{2^n}$ exactly.

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9. Find the fifth partial sum $T_5(x)$ of the Taylor series for $f(x) = \sin x$ about a centre $x = \frac{\pi}{4}$ and find an upper bound to the error caused by replacing $\sin x$ by $T_5(x)$ for $0 < x < \frac{\pi}{2}$.

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10. Find the Maclaurin series expansion for $y = \tan x$ as far as the term in x^4.

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11. Find the Maclaurin series for $f(x) = \ln(1+x)$ and find its interval of convergence.

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12. Find the Fourier series for $f(x) = 4 - x^2 \quad (-2 \le x \le 2)$

13. Find the Fourier sine series for $f(x) = x - x^2 \quad (0 \le x \le 1)$

14. Find the Fourier cosine series for $f(x) = \begin{cases} 1-x & (0 \le x < 1) \\ 0 & (1 \le x \le 2) \end{cases}$.

☜ Back to the index of questions On to the solutions ☞