1. Review of Calculus

We begin this course with a refresher on differentiation and integration from MATH 1000 and MATH 1001.

1.1 Reminder of some Derivatives (review from MATH 1000)

Product Rule:

$$\frac{d}{dx}(u \cdot v) =$$

Quotient Rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right) =$$

Chain Rule:

If $y = f(u)$ and $u = g(x)$ then

$$\frac{d}{dx}\left(x^n\right) =$$

$$\frac{d}{dx}\left(e^{kx}\right) =$$

$$\frac{d}{dx}(\ln x) =$$

$$\frac{d}{dx}(\sin x) =$$

$$\frac{d}{dx}(\cos x) =$$

$$\frac{d}{dx}(\tan x) =$$

$$\frac{d}{dx}(\csc x) =$$

$$\frac{d}{dx}(\sec x) =$$

$$\frac{d}{dx}(\cot x) =$$

$$\frac{d}{dx}\left(e^{u(x)}\right) =$$

$$\frac{d}{dx}(u(x))^n =$$

$$\frac{d}{dx}\sin^n(u(x)) =$$

$$\frac{d}{dx}\cos^n(u(x)) =$$

$$\frac{d}{dx}\left(a^{u(x)}\right) =$$

$$\frac{d}{dx}\left(\ln\left(u(x)\right)\right) =$$

Algebra of exponents:

$$e^u \cdot e^v = \qquad\qquad \left(e^u\right)^v =$$

Algebra of logarithms:

$$\ln\left(u \cdot v\right) = \qquad\qquad \ln\left(\frac{u}{v}\right) =$$

$$\ln\left(x^n\right) =$$

$$\cos^2\theta + \sin^2\theta =$$

Double angle formulae:

$$\sin\left(2\theta\right) =$$

$$\cos\left(2\theta\right) =$$

Implicit Differentiation

Example 1.1.1

Show that $\frac{d}{dx}\left(a^{u(x)}\right) = u'(x)\, a^{u(x)} \ln a$

1.2 Reminder of some Integrals (review from MATH 1001)

$$\int u'(x)\cdot\left(u(x)\right)^n dx =$$

Example 1.2.1

$$\int \tan^4 x \cdot \sec^2 x \, dx =$$

$$\int \frac{u'(x)}{u(x)} dx =$$

Example 1.2.2

$$\int \tan x \, dx =$$

Example 1.2.3

$$\int \frac{1}{x \ln x} dx =$$

$$\int u'(x)\, e^{u(x)} dx =$$

Example 1.2.4

$$\int x^2 e^{x^3} dx =$$

$$\int u'(x)\cos\big(u(x)\big)\,dx =$$

Example 1.2.5

$$\int e^{x}\cos\left(e^{x}\right)dx =$$

Some trigonometric integrals:

$$\int \sin^2\theta\, d\theta =$$

$$\int \cos^2\theta\, d\theta =$$

$$\int \sec^2 x\, dx =$$

$$\int \sec x \tan x\, dx =$$

$$\int \sec x\, dx =$$

$$\int \csc^2 x\, dx =$$

$$\int \csc x \cot x\, dx =$$

$$\int \csc x\, dx =$$

$$\int \frac{1}{a^2 + x^2}\, dx$$

$$\Rightarrow \quad \int \frac{1}{a^2 + x^2}\, dx =$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\, dx$$

$$\Rightarrow \quad \int \frac{1}{\sqrt{a^2 - x^2}}\, dx =$$

Integration by parts (tabular version) – the table terminates in one of three ways:

1. Left column ends in zero (one factor in the integrand must be a polynomial).

Example 1.2.6

$$\int \left(x^2 - 1\right) \cos 2x \, dx =$$

2. The last row is easily integrated:

Example 1.2.7

$$\int x^n \ln x \, dx =$$

3. The last row is a constant multiple of the original integrand:

Example 1.2.8

$$I = \int e^{ax} \sin bx \, dx =$$

ENGI 3425 assumes mastery of the concepts and techniques in MATH 1000, 1001 and 2050.

1.3 Hyperbolic Functions

When a uniform inelastic (unstretchable) perfectly flexible cable is suspended between two fixed points, it will hang, under its own weight, in the shape of a catenary curve. The equation of the standard catenary curve is most concisely expressed as the hyperbolic cosine function, $y = \cosh x$, where

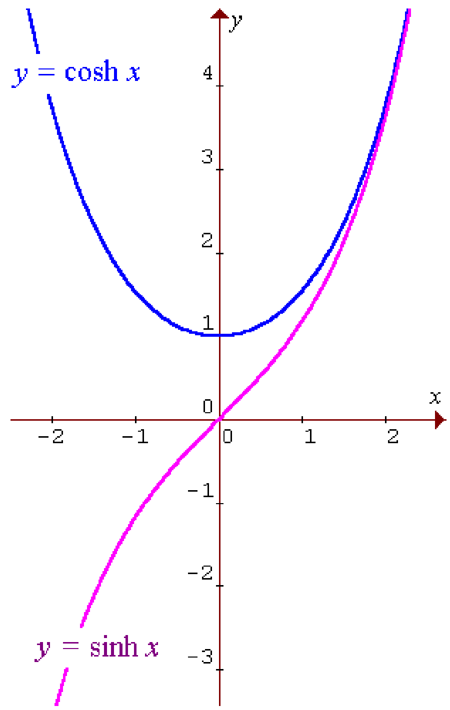
$$\cosh x = \frac{e^x + e^{-x}}{2}$$

The solutions to some differential equations can be expressed conveniently in terms of hyperbolic functions.

Another hyperbolic function is

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

The graphs of these two hyperbolic functions are displayed here:



The other four hyperbolic functions are

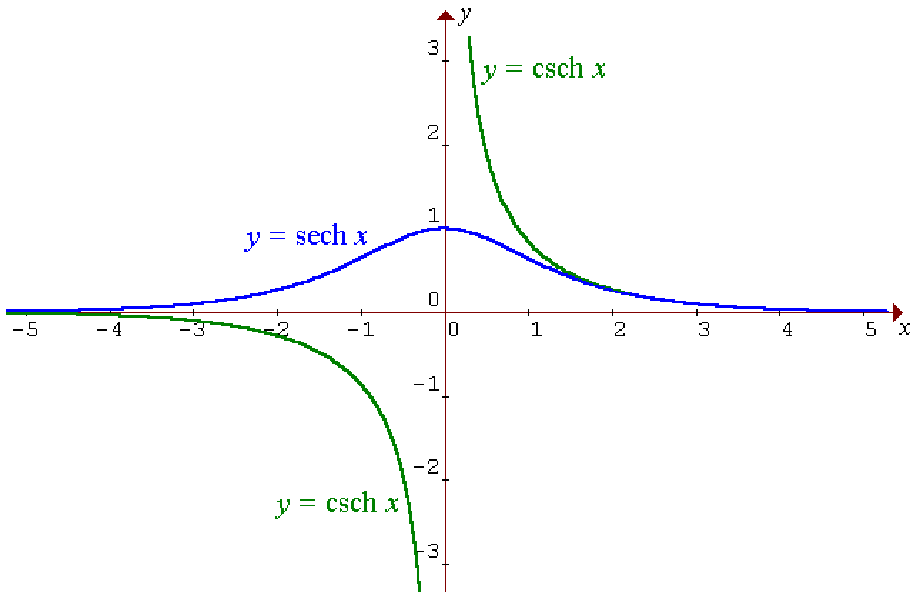
$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}, \qquad \operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}},$$

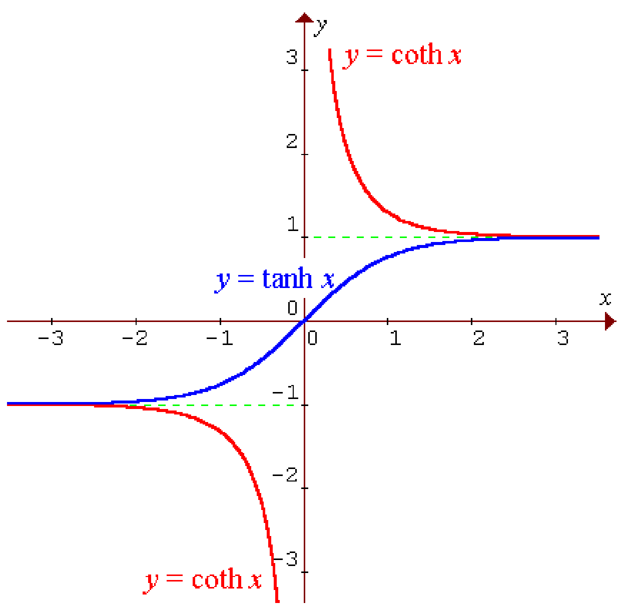
$$\coth x = \frac{1}{\tanh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}} \quad \text{and} \quad \operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}.$$

Unlike the trigonometric functions, the hyperbolic functions are not periodic.
However, parity is preserved:
Of the six trigonometric function, only $\cos\theta$ and $\sec\theta$ are even functions.
Of the six hyperbolic functions, only $\cosh x$ and $\operatorname{sech} x$ are even functions.

The other four trigonometric functions and the other four hyperbolic functions are all odd.

Graphs of the other four hyperbolic functions:





The solutions to some ordinary differential equations that model logistic growth (constrained growth) of a population resemble the hyperbolic tangent function.

There is a close relationship between the hyperbolic and trigonometric functions.

From the Euler form for $e^{j\theta}$, $e^{j\theta} = \cos\theta + j\sin\theta$, (where $j = \sqrt{-1}$)

Identities:

Let $x = j\theta$:

$$\sin^2\theta + \cos^2\theta \equiv 1 \quad \Rightarrow$$

$$1 + \tan^2\theta \equiv \sec^2\theta \quad \Rightarrow$$

Derivatives

$$\frac{d}{dx}(\sinh x) =$$

$$\frac{d}{dx}(\cosh x) =$$

$$\frac{d}{dx}(\tanh x) =$$

$$\frac{d}{dx}(\operatorname{csch} x) =$$

End of Chapter 1