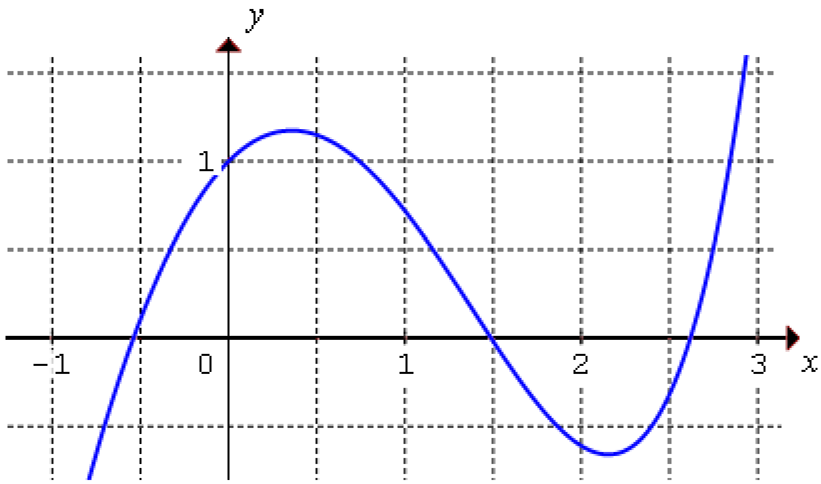
ENGI 4430
Mid Term Test
2019 June 19

1. The equation $f(x) = 0$, where $f(x) = e^x - 2x^2$, has three real roots (solutions), as illustrated in this graph.



(a) Use Newton's method, with an appropriate initial estimate x_0, to determine the value of the **middle** root, correct to four significant figures. [11]

(b) Why should an initial estimate near $x_0 = 2.2$ not be used? [3]

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2. A particle is following a path given by

$$\vec{\mathbf{r}}(t) = \begin{bmatrix} 4\cos t \\ 3 \\ 4\sin t \end{bmatrix}$$

where t is the time $(t \geq 0)$ in seconds and distances are measured in metres.

(a) By any valid method, find all of the following [13]

- the tangential component of acceleration a_T
- the normal component of acceleration a_N
- the curvature κ
- the unit principal normal vector $\hat{\mathbf{N}}$

(b) Describe geometrically the path that the particle is following. [3]

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3. For the curve whose equation in polar coordinates is $r = 1 + 3\cos\theta$

(a) Find the values of θ (in the range $-\pi < \theta \le \pi$) at which r attains its maximum and minimum values. [3]

(b) To the nearest degree, find the acute angle α that the polar tangents make with the horizontal axis. [3]

(c) Sketch the curve. Identify the region where $r < 0$. [14]

[Cartesian and polar grids were provided with the question paper.]

BONUS QUESTION

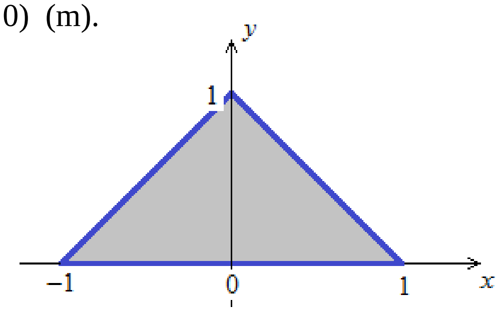
4. A thin plate D has the shape of a triangle on the x-y plane whose vertices are the points (–1, 0), (0, 1) and (1, 0) (m). [+5]

[This is the same region as in Quiz #2.]

The surface density everywhere on D is now

$$\sigma(x, y) = ay + b \quad \left(\text{kg m}^{-2}\right)$$

where a , b are constants, such that the density is non-negative everywhere on D.



Find the maximum and minimum possible values of $\bar{y}$ (the y coordinate of the centre of mass).

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