ENGI 4430

Mid Term Test

2020 June 17

1. Part of the hyperbola $y^2 = x^2 - 16$ in the first quadrant, between $x = 4$ and $x = 5$, is rotated once around the *x*-axis to form a surface of revolution (which is part of a surface known as a "hyperboloid of two sheets").

   (a) Write down the equation of this surface of revolution. [3]

   (b) Show that the curved surface area of this surface of revolution is [7]

$$A = 2\pi \int_4^5 \sqrt{2x^2 - 16}\, dx$$

   (c) Use Simpson's rule with $n = 4$ intervals to estimate the value of A to three significant figures. [10]

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2. A region D is bounded by the parabola $y^2 = x$ and the line $x = 1$, as shown. [20]
   The surface density σ at any point $P(x, y)$ in D is directly proportional to the distance of P from the line $x = 1$.
   Find the exact location $(\bar{x}, \bar{y})$ of the centre of mass of D.

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3. The location $\vec{\mathbf{r}}(t)$ of a particle at any time t is given (in SI units) by

$$\vec{\mathbf{r}}(t) = 6t\,\hat{\mathbf{i}} + 8t\,\hat{\mathbf{j}} - 5t^2\hat{\mathbf{k}}$$

   (a) Show that the magnitude of the acceleration vector is $a = 10\ \text{ms}^{-2}$. [6]

   (b) Find the radius of curvature $\rho(t)$. [6]

   (c) Find the tangential **and** normal components of the acceleration vector. [8]

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4. *BONUS QUESTION* [+5]

   Find the location (in either polar or Cartesian coordinates) of all distinct vertical tangents to the curve whose equation in plane polar coordinates is $r = 2 + \cos\theta$ **and** sketch the graph. {A polar grid was provided with the question paper.]

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