ENGI 4430 Advanced Calculus for Engineering
Faculty of Engineering and Applied Science

Problem Set 10 Questions

[Partial differential equations]

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1. Is the function $f(x, y) = 3x^2 y - y^3$ harmonic and, if so, on what domain?

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2. Find the subsequent motion of an infinite string that is released from rest with the initial displacement

$$\phi(x) = \frac{1}{1+8x^2}$$

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3. Classify the partial differential equation $\dfrac{\partial^2 u}{\partial x^2} - 6\dfrac{\partial^2 u}{\partial x\, \partial y} + 8\dfrac{\partial^2 u}{\partial y^2} = 0$ and find its complete solution, given the additional information $u(x,0) = 8x^3$ and $u_y(x,0) = 12x^2$.

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4. Classify the partial differential equation $4\dfrac{\partial^2 u}{\partial x^2} + 12\dfrac{\partial^2 u}{\partial x\, \partial y} + 9\dfrac{\partial^2 u}{\partial y^2} = 78$ and find its complete solution, given the additional information $u(0, y) = 3y^2$ and $u(x,0) = 0$.

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5. Classify the partial differential equation $\dfrac{\partial u}{\partial t} = 4\dfrac{\partial^2 u}{\partial x^2}$ and find its complete solution on the interval $0 \le x \le 100$ for all positive time *t*, given the additional information

$u(0,t) = 0$ and $u(100,t) = 100 \quad \forall t \ge 0$

and $u(x,0) = 2x - \left(\dfrac{x}{10}\right)^2 \quad \forall x \in [0, 100]$

Also write down the steady state solution.

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6. Classify the partial differential equation $4\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial x\,\partial y} + \frac{\partial^2 u}{\partial y^2} = 0$ and find its general solution.

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7. Classify the partial differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x\,\partial y} + \frac{\partial^2 u}{\partial y^2} = 0$ and find its general solution.

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8. Classify the partial differential equation $\frac{\partial^2 u}{\partial x^2} + 3\frac{\partial^2 u}{\partial x\,\partial y} + 2\frac{\partial^2 u}{\partial y^2} = -6$
and find its complete solution, given the additional information
$u(x,0) = -2x$ and $u_y(x,0) = 1-2x$.
Also verify that your solution satisfies the PDE and both conditions.

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9. Classify the partial differential equation $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 6y$
and find its complete solution, given the additional information
$u(0,y) = y^3$ and $u_x(0,y) = -3y^2$.
Also verify that your solution satisfies the PDE and both conditions.

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10. An ideal perfectly elastic string of length 1 m is fixed at both ends (at $x = 0$ and at $x = 1$). The string is displaced into the form $y(x,0) = f(x) = x^2(1-x)^2$ and is released from rest. Waves travel without friction along the string at a speed of 2 m/s. Find the displacement $y(x, t)$ at all locations on the string ($0 < x < 1$) and at all subsequent times ($t > 0$).

Write down the complete Fourier series solution and the first two non-zero terms.

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11. The transverse displacement $y(x,t)$ of a point at time t and location x on an ideal wire whose ends are fixed at $x = 0$ and $x = 1$ is governed by the partial differential equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

At time $t = 0$ the displacement of the wire is $y(x,0) = \sin(3\pi x)$ and it is released from rest. Find the displacement $y(x, t)$ at all subsequent times.

[You may quote $\int_0^L \sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi x}{L}\right) dx = \begin{cases} \frac{L}{2} & (m = n) \\ 0 & (m \neq n) \end{cases}$

for all positive integers m, n.]

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12. The transverse displacement $y(x, t)$ of a point at time t and location x on an ideal wire whose ends are fixed at $x = 0$ and $x = 4$ is governed by the partial differential equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}, \quad \text{with } c = 2.$$

At time $t = 0$ the displacement of the wire is $y(x,0) = 2x - \frac{1}{2}x^2$ and it is released from rest. Find the displacement $y(x, t)$ at all subsequent times. Write down the general term of the series for $y(x, t)$ ***and*** write down the first three non-zero terms.

☜ Back to the index of questions On to the solutions to this problem set ☞