ENGI 4430 Advanced Calculus for Engineering
Faculty of Engineering and Applied Science

Problem Set 4 Questions

[Numerical Methods]

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1. Use Simpson's rule with four intervals to estimate $I = \int_0^{\pi} \sqrt{\sin\theta}\, d\theta$

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2. Use the trapezoidal rule with six intervals to estimate $I = \int_0^1 \frac{1}{1+x^2}\, dx$.
Use Simpson's rule with six intervals to estimate I.
Find the exact value of I and comment on the accuracy of the two approximations.

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3. Use Simpson's rule with eight intervals to estimate $I = \int_0^1 e^{-x^2}\, dx$.
[The "error function" is defined to be $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$. It has multiple uses, including the calculation of probabilities for random quantities that follow a normal distribution. It cannot be evaluated exactly, except for a few special choices of x.]

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4. In Problem Set 2 Question 3, the total arc length along any one loop of the curve $r^2 = 4\cos 3\theta$ was found to be $L = 2\int_0^{\pi/6} \sqrt{9\sec 3\theta - 5\cos 3\theta}\, d\theta$. Explain why a simple trapezoidal or Simpson's rule can ***not*** be used to estimate this arc length.

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5. Use Newton's method (and confirm with a graphical method) to find the value of $\tanh^{-1}\left(\frac{1}{2}\right)$ to six significant figures, where $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{1-e^{-2x}}{1+e^{-2x}}$.

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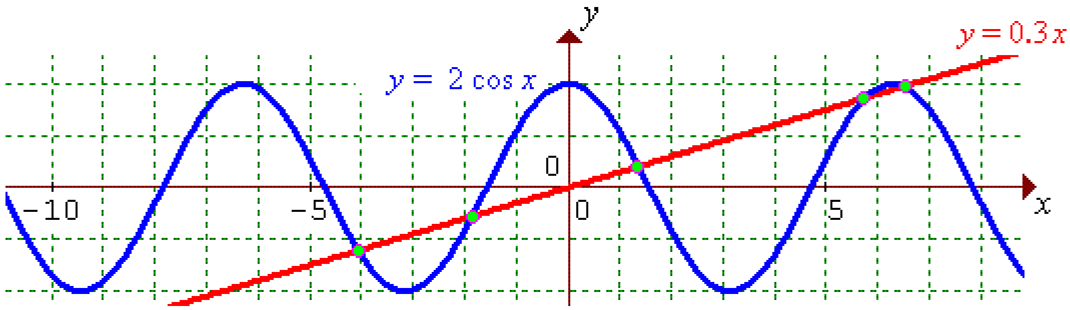
6. As seen in Question 5, when Newton's method works well, the convergence can be quite rapid. Now consider the solutions to $2\cos x = 0.3x$. As seen from the plot below, there are five solutions to this equation.



Let us try to find the largest root.

(a) Use Newton's method with an initial guess of $x_0 = 7$ to find a solution correct to five decimal places.

(b) Use Newton's method with an initial guess of $x_0 = 6$ to find a solution to 5 d.p.

(c) Use Newton's method with an initial guess of $x_0 = 6.1000$ to find a solution to 5 d.p.

(d) Use Newton's method with an initial guess of $x_0 = 6.0999$ to find a solution to 5 d.p.

(e) Comment briefly on these different results. What is the single greatest factor in the variability of these results?

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7. This question provides some practice in algebra and refers to page 5.04 of the lecture notes. From the definition $\sinh x \equiv \dfrac{e^x - e^{-x}}{2}$, verify that

$$\sinh^{-1} x \equiv \ln\left(x + \sqrt{1+x^2}\right) \equiv -\ln\left(\sqrt{1+x^2} - x\right)$$

☜ Back to the index of questions On to the solutions to this problem set ☞