4. Lines of Force

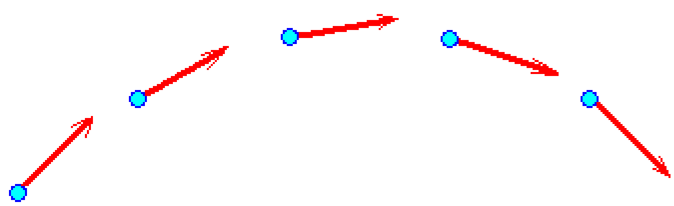
A vector function of n variables in $\mathbb{R}^n$ is a **vector field**.

$$\vec{\mathbf{F}}(x,y,z) = \begin{bmatrix} f_1(x,y,z) \\ f_2(x,y,z) \\ f_3(x,y,z) \end{bmatrix}$$

(and the f_i form scalar fields).

The domain must be defined. If not explicitly mentioned, the domain is assumed to be all of that part of $\mathbb{R}^n$ for which all of the scalar fields f_i are defined.

A vector field defines a vector $\vec{\mathbf{F}}$ at each point in the domain.
If these vectors are tangents to a family of curves, then those curves are **streamlines** or **flow lines** or **lines of force**.



Let $\vec{\mathbf{F}}(x,y,z)$ be a vector field to a family of lines of force $\vec{\mathbf{r}}(x,y,z)$. Then

Example 4.01

Find the lines of force associated with the vector field $\mathbf{F} = \begin{bmatrix} e^z & 0 & -x^2 \end{bmatrix}^{\mathrm{T}}$ and find the line of force that passes through the point (4, 2, 0).

Example 4.02

Find the lines of force associated with the vector field $\mathbf{F} = \begin{bmatrix} x^2 & 2y & -1 \end{bmatrix}^{\mathrm{T}}$
and find the line of force that passes through the point (–1, 6, 2).

Example 4.03

Find the streamlines associated with the velocity field $\vec{\mathbf{v}} = \begin{bmatrix} \dfrac{-y}{x^2 + y^2} & \dfrac{x}{x^2 + y^2} & 0 \end{bmatrix}^{\mathrm{T}}$ and find the streamline through the point (1, 0, 0).

[End of Chapter 4]