

**Some standard integrals**

$$\int x^n dx =$$

$$\int (f(x))^n f'(x) dx =$$

Example:  $\int \sin^5 x \cos x dx =$

$n = -1$  case:

Examples:  $\int \tan x dx =$

$$\int \cot x dx =$$

$$\int f'(x) e^{f(x)} dx =$$

Examples:  $\int e^{kx} dx =$

$$\int x^2 e^{x^3} dx =$$

$$\int f'(x) \cos(f(x)) dx =$$

Example:  $\int \cos(kx) dx =$

$$\int f'(x) \sin(f(x)) dx =$$

Example:  $\int \sin(kx) dx =$

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Other trigonometric integrals:

$$\int \sec^2 x \, dx = \quad \int \sec x \tan x \, dx = \quad \int \sec x \, dx =$$

$$\int \csc^2 x \, dx = \quad \int \csc x \cot x \, dx = \quad \int \csc x \, dx =$$

Using the identity  $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$  (which can be derived from  $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ ),

$$\int \sin^2(kx) \, dx =$$

Using the identity  $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$ ,

$$\int \cos^2(kx) \, dx =$$

Using the substitution  $x = a \tan \theta \Rightarrow dx = a \sec^2 \theta \, d\theta$  and

$$a^2 + x^2 = a^2(1 + \tan^2 \theta) = a^2 \sec^2 \theta,$$

$$\int \frac{1}{a^2 + x^2} \, dx =$$

Using the substitution  $x = a \sin \theta \Rightarrow dx = a \cos \theta \, d\theta$  and

$$\sqrt{a^2 - x^2} = \sqrt{a^2(1 - \sin^2 \theta)} = a \cos \theta,$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \, dx =$$

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**Integration by parts** (tabular version) – the table terminates in one of three ways:

1. Left column ends in zero (one factor in the integrand must be a polynomial).

Example:

$$\int (x^2 - 1) \cos 2x \, dx =$$

2. The last row is easily integrated:

Example:  $(n \neq -1)$

$$\int x^n \ln x \, dx =$$

Case  $n = 0$ :  $\int \ln x \, dx =$

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3. The last row is a constant multiple of the original integrand:

Example:

$$I = \int e^{ax} \sin bx \, dx =$$

Example:

$$I = \int \frac{\ln x}{x} \, dx =$$

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