ENGI 9420 Engineering Analysis
Assignment 1 Questions

2012 Fall

due in class 2012 September 17 Monday

[First order ODEs, Sections 1.01-1.05]

Note*:** In this assignment, do ***not use Laplace transform methods at all.

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1. For the initial value problem

$$\frac{dy}{dx} + 2y = 2, \qquad y(0) = 4$$

(a) Classify the ODE (as one or more of separable, exact, linear, or Bernoulli). [2]
(b) Obtain the complete solution by two different methods. [10]
(c) Verify that your solution does satisfy the initial value problem. [4]

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2. For the initial value problem

$$\frac{dy}{dx} + 2y = 2y^3, \qquad y(0) = 4$$

(a) Classify the ODE (as one or more of separable, exact, linear, or Bernoulli). [2]
(b) Obtain the complete solution. [5]
(c) Verify that your solution does satisfy the initial value problem. [4]
(d) Find the complete solution when the initial condition is replaced by $y(0) = 0$. [4]

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3. For the ordinary differential equation

$$y\,dx + (2x + 3y)\,dy = 0$$

(a) Show that the ODE is not exact. [2]
(b) Find an integrating factor for this ODE. [4]
(c) Hence find the general solution (in implicit form). [6]

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4. Find the complete solution to the initial value problem [12]

$$\frac{dy}{dx} + 3y = 3y^{2/3}, \quad y(0) = 0$$

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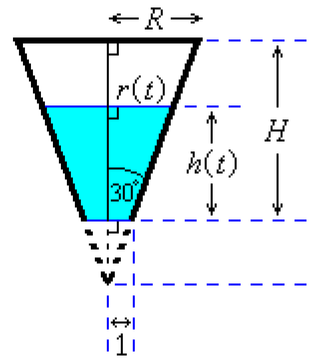
5. A conical tank, of half-angle 30°, contains liquid as shown. The apex has been cut off to leave a circular hole of radius 1 centimetre, through which liquid drains out from the container. The "head" (or height) of liquid above the hole at any instant t is $h(t)$. The tank has a radius at the top of R and a height from the top to the hole of H. All distances are measured in centimetres.



The rate at which the volume $V(t)$ of liquid in the tank changes due to liquid draining at discharge speed $v(t)$ through a hole of area A is given by the differential equation

[20]

$$\frac{dV}{dt} = -kAv$$

where k is an experimentally determined constant, (dependent on viscosity and the geometry of the opening), between 0 and 1. For this question, assume $k = 0.7$.

In addition, Toricelli's law (equating the gain of kinetic energy of every point in the water to the loss of gravitational potential energy of that point) leads to

$$v(t) = \sqrt{2g\,h(t)}$$

Find how long it takes (T) for a full tank to drain completely, as a function of the truncated height H of the cone.

Hence find the value of T to the nearest second when $H = 30$ cm.
Take $g = 981\,\text{cm}\,\text{s}^{-2}$.

6. A five metre long chain with a constant line density of ρ kg/m is supported in a pile on a platform several metres above the floor of a warehouse. It is wound around a frictionless pulley at the edge of the platform, with one metre of chain already hanging down at time $t = 0$, when the chain is released from rest. Let $x(t)$ represent the length of that part of the chain hanging down from the pulley at time t and let $v(t)$ be the speed of that part of the chain at time t.

(a) Show that the ordinary differential equation governing the speed of the chain is [8]

$$\frac{dv}{dx} + \frac{1}{x} v = \frac{g}{v}, \qquad (1 \le x \le 5)$$

where $g \approx 9.81 \, \text{m s}^{-2}$ is the acceleration due to gravity and where all frictional forces are ignored.

(b) Determine the speed with which the trailing end of the chain leaves the pulley. [17]

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