ENGI 9420 Engineering Analysis
Assignment 2 Questions

2012 Fall
due in class 2012 September 24 Monday
[Second order ODEs, Laplace transforms; Sections 1.01-1.09]

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1. Use Laplace transforms to solve the initial value problem [10]

$$\frac{dy}{dx} + 2y = 2\,, \qquad y(0) = 4$$

[This was Question 1 on Assignment 1]

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2. Find the complete solution of the initial value problem

$$\frac{d^2 y}{dx^2} + 7\frac{dy}{dx} + 10y = 6e^{-2x}\,, \qquad y(0) = 1\,,\; y'(0) = -3$$

(a) without using Laplace transforms; and [8]
(b) using Laplace transforms. [8]

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3. An underdamped mass-spring system, with an oscillating force applied, is modelled by the ordinary differential equation

$$\frac{d^2 x}{dt^2} + 2\frac{dx}{dt} + 2x = 10\sin t$$

Find the general solution $x(t)$

(a) without using Laplace transforms; and [6]
(b) using Laplace transforms. [10]

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4. Find the general solution of the ordinary differential equation [10]

$$\frac{d^2 y}{dx^2} + y = \sec x\,, \qquad \left(0 \le x < \frac{\pi}{2}\right)$$

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5. A mass-spring system is at rest until it is struck by a hammer at time $t = 4$ (seconds). [10]
The response $x(t)$ is modelled by the initial value problem

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 53x = 21\,\delta(t-4), \qquad x(0) = x'(0) = 0$$

where $\delta(t-a)$ is the Dirac delta function.
Use Laplace transforms to find the complete solution of this initial value problem.

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6. Find the Laplace transform $F(s)$ of [10]

$$f(t) = t\,e^{-2t}\cos 3t$$

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7. Find the function $f(t)$ whose Laplace transform is [12]

$$F(s) = \frac{48}{(s+2)(s^2+4s+20)}$$

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8. Use the integration property of Laplace transforms, $\mathcal{L}^{-1}\left\{\frac{F(s)}{s}\right\} = \int_0^t \mathcal{L}^{-1}\{F(s)\}\,d\tau$

twice, in order to establish $\mathcal{L}^{-1}\left\{\frac{1}{s^2(s^2+\omega^2)}\right\} = \frac{\omega t - \sin\omega t}{\omega^3}$ [8]

and confirm this result using partial fractions. [8]

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