ENGI 9420 Engineering Analysis
Assignment 3 Questions

2012 Fall

due in class 2012 October 10 (Wednesday)

[Series solution of ODEs, matrix algebra; numerical methods; Chapters 1, 2 and 3]

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1. Find a power series solution about $x = 0$, as far as the term in x^7, to the ordinary differential equation [12]

$$\frac{d^2 y}{dx^2} + y = \sec x\,, \qquad \left(0 \le x < \frac{\pi}{2}\right)$$

[This ODE is also in Assignment 2 Question 4.]

You may quote the Maclaurin series expansion for $\sec x$:

$$\sec x = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \frac{61x^6}{720} + \frac{277x^8}{8064} + \ldots \qquad \left(|x| < \tfrac{\pi}{2}\right)$$

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2. Use the method of Frobenius to find the general solution of the ordinary differential equation [12]

$$x^2 \frac{d^2 y}{dx^2} + \tfrac{1}{2} x(1+4x)\frac{dy}{dx} + \left(x - \tfrac{1}{2}\right) y = 0$$

as a series about $x = 0$, as far as the fourth non-zero term.

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3. Find the entire series solution (using the method of Frobenius, adapted to a first order ODE) about $x = 0$ of the ordinary differential equation [8]

$$\frac{dy}{dx} + \frac{4y}{x} = 2$$

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4. In Chapter 2 of the lecture notes, dimensional analysis is used to derive the functional form of the Planck length L_P in terms of the universal constants G, h and c. [8]

In the study of the steady flow of incompressible fluid through a pipe, it is known that the volume of liquid issuing per second from a pipe, Q, depends on the coefficient of viscosity η (measured in the units kg $m^{-1}s^{-1}$), the radius a of the pipe (measured in metres) and the pressure gradient p/l set up along the pipe (measured in the units N $m^{-2}m^{-1}$ or, equivalently, kg $m^{-2}s^{-2}$).

Use a similar dimensional analysis to derive the functional form for Q in terms of η, a and p/l.

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5. Find the intersection of the three planes [8]

$$\begin{aligned} x + 2y + 3z \quad &= 0 \\ 2x + 5y + 2z + 1 &= 0 \\ x + 2y + 4z - 1 &= 0 \end{aligned}$$

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6. By reducing the appropriate linear system to echelon form, show that the intersection of the three planes [8]

$$\begin{aligned} x + y + z + 1 &= 0 \\ x + 2y + 3z + 4 &= 0 \\ 4x + 3y + 2z + 1 &= 0 \end{aligned}$$

is the line $\dfrac{x-2}{1} = \dfrac{y-(-3)}{-2} = \dfrac{z-0}{1}$.

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7. Find the value of the determinant and, if it exists, the inverse matrix, for the matrix [12]

$$\mathrm{A} = \begin{bmatrix} 1 & 2 & 0 & -1 \\ \frac{1}{2} & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{1}{2} \\ -1 & 0 & 2 & 1 \end{bmatrix}$$

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8. In Assignment 1, Question 5, find, correct to the nearest millimetre, the initial head H of water needed in the conical tank in order for the tank to drain completely in exactly one minute, using

(a) the method of bisection (***or*** a graphical zoom-in, in which case provide sketches or screenshots) and explain your choice of initial values. [6]

(b) Newton's method and explain your choice of initial value. [6]

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9. Why should Newton's Method **not** be used to find a root of $e^x = \tan x$, [8]
(except when the initial guess is very near the true value)?

Demonstrate the problem by using Newton's Method to try to find the first positive root, with initial guesses of 0.99 and 1.00.

10. Use the standard fourth order Runge-Kutta method, with a step size of $h = 0.2$, to find the value at $x = 0.6$ of the solution of the initial value problem [12]

$$\frac{dy}{dx} = x^3\left(y^2 - y\right), \qquad y(0) = 0.2$$

Show your calculations for the constants k_1, k_2, k_3, k_4 and for y_1 in the first iteration. You may submit output from a spreadsheet program (such as Excel) for the other steps.

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