ENGI 9420 Engineering Analysis
Additional Exercises

2012 Fall
[not to be handed in]
[Partial differential equations; Chapter 8]

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1 The function $u(x,y)$ satisfies $\dfrac{\partial^2 u}{\partial x^2} - 3\dfrac{\partial^2 u}{\partial x\,\partial y} + 2\dfrac{\partial^2 u}{\partial y^2} = 0$, subject to the boundary conditions $u(x,0) = \left.\dfrac{\partial}{\partial y}u(x,y)\right|_{y=0} = -1$. Classify the partial differential equation (hyperbolic, parabolic or elliptic) and find the complete solution $u(x,y)$.

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2. Classify the partial differential equation $4\dfrac{\partial^2 u}{\partial x^2} + 12\dfrac{\partial^2 u}{\partial x\,\partial y} + 9\dfrac{\partial^2 u}{\partial y^2} = 0$ and find its general solution.

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3. A disturbance on a very long string causes a vertical displacement $y(x,t)$ at a distance x from the origin at time t. The string is released from rest at time $t = 0$ with initial displacement $f(x) = \dfrac{1}{1+8x^2}$.

(a) Find the subsequent motion of this string $y(x,t)$.

(b) Sketch or plot the wave form at time $t = 0$ and at any two subsequent times.

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4. Classify the partial differential equation $\nabla^2 u = \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 12x$ and find the complete solution given the additional information

$u(0,y) = 0$, $\left.\left(\dfrac{\partial}{\partial x}u(x,y)\right)\right|_{x=0} = 3y^2$

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5. Classify the partial differential equation $\dfrac{\partial^2 u}{\partial^2 x} - 7\dfrac{\partial^2 u}{\partial x\,\partial y} + 10\dfrac{\partial^2 u}{\partial^2 y} = -3$

and find the complete solution, given the additional information

$u(x,0) = 2x^2 + 4\,, \qquad u_y(x,0) = x$

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6. Classify the partial differential equation $\dfrac{\partial u}{\partial t} = 4\dfrac{\partial^2 u}{\partial x^2}$ and find its complete solution on the interval $0 \le x \le 100$ for all positive time t, given the additional information

$u(0,t) = 0$ and $u(100,t) = 100 \quad \forall t \ge 0$

and $u(x,0) = 2x - \left(\dfrac{x}{10}\right)^2 \quad \forall x \in [0,100]$

Also write down the steady state solution.

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7. An ideal perfectly elastic string of length 1 m is fixed at both ends (at $x = 0$ and at $x = 1$). The string is displaced into the form $y(x,0) = f(x) = x^2(1-x)^2$ and is released from rest. Waves travel without friction along the string at a speed of 2 m/s. Find the displacement $y(x, t)$ at all locations on the string ($0 < x < 1$) and at all subsequent times ($t > 0$).

Write down the complete Fourier series solution and the first two non-zero terms.

☜ Return to the index of assignments On to the solutions ☞