Trigonometric identities needed in examples 7.01.1 & 7.02.1
(pages 7.04 and 7.05 of the lecture notes)

From page 7.04 of the lecture notes:

$$\int_{-\pi}^{\pi} \sin mx \sin nx \, dx = ?$$

Recall that $\cos(A+B) = \cos A \cos B - \sin A \sin B$ **(A)**

and $\cos(A-B) = \cos A \cos B + \sin A \sin B$ **(B)**

(B) – (A) $\Rightarrow \cos(A-B) - \cos(A+B) = 0 + 2\sin A \sin B$

$$\Rightarrow \sin A \sin B = \frac{\cos(A-B) - \cos(A+B)}{2}$$

$$\Rightarrow \int_{-\pi}^{\pi} \sin mx \sin nx \, dx = \frac{1}{2}\int_{-\pi}^{\pi} \left(\cos(m-n)x - \cos(m+n)x\right) dx$$

which is now integrated easily, as is

$$\int_{-L}^{L} \sin\left(\frac{m\pi x}{L}\right) \sin\left(\frac{n\pi x}{L}\right) dx = \frac{1}{2}\int_{-L}^{L} \left(\cos\frac{(m-n)\pi x}{L} - \cos\frac{(m+n)\pi x}{L}\right) dx$$

on page 7.05.

(B) + (A) $\Rightarrow \cos(A-B) + \cos(A+B) = 2\cos A \cos B + 0$

$$\Rightarrow \cos A \cos B = \frac{\cos(A-B) + \cos(A+B)}{2}$$

$$\Rightarrow \int_{-L}^{L} \cos\left(\frac{m\pi x}{L}\right) \cos\left(\frac{n\pi x}{L}\right) dx = \frac{1}{2}\int_{-L}^{L} \left(\cos\frac{(m+n)\pi x}{L} + \cos\frac{(m-n)\pi x}{L}\right) dx$$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$ **(C)**

$\sin(A-B) = \sin A \cos B - \cos A \sin B$ **(D)**

(C) + (D) $\Rightarrow \sin(A+B) + \sin(A-B) = 2\sin A \cos B + 0$

$$\Rightarrow \sin A \cos B = \frac{\sin(A+B) + \sin(A-B)}{2}$$

$$\Rightarrow \int_{-L}^{L} \sin\left(\frac{m\pi x}{L}\right) \cos\left(\frac{n\pi x}{L}\right) dx = \frac{1}{2}\int_{-L}^{L} \left(\sin\frac{(m+n)\pi x}{L} + \sin\frac{(m-n)\pi x}{L}\right) dx$$