From Close, Frederick and Newell, and Karnopp, Margolis and Rosenberg:

**Fluid Elements**

Hydraulic systems: typically incompressible flow  
Acoustic systems: compressible flow, such as air or other gas  
Pressure: normally defined as “gauge pressure”

**Capacitance (potential energy storage)**

Open tank:

\[ P_3 = \Phi_C^{-1}(V_3) \quad V_3 = \int_0^t Q_3\,dt \]

Rigid pipe segment with compressible fluid:

\[ \Delta P_3 = \frac{B}{V_0}\Delta V_3 \quad C_3 = \frac{V_0}{B} \]

\[ \Delta P_3 = \rho_0c^2 V_3 / V_0 \quad C_3 = V_0 / \rho_0c^2 \]

Elastic pipe segment:

\[ \Delta P_3 = \left( t_wE / 2r_0V_0 \right) \Delta V_3 \quad C_3 = 2r_0V_0/t_wE \]
Compressed gas accumulator

\[ P_3 = \frac{P_0 V_0^\gamma}{(V_0 - V_3)^\gamma} \]

**Resistance (Dissipation)**

\[ P_3 = \Phi_R(Q_3) \]

\[ P_3 = R_3 Q_3 \]
\[ Q_3 = P_3 / R_3 \]

Sources of fluid resistance (friction, dissipation)
- restrictions and orifices
- bends in pipe
- friction between fluid and pipe walls

Typically nonlinear relation between pressure and fluid flow.

The following figures show:

- porous plug

\[ P_3 = R_3 Q_3 \]

- long pipe with wall friction

\[ P_3 = R_3 Q_3 \]

\[ R_3 = 128 \mu l / \pi d^4 \]

\[ \text{Re} = 4 \rho Q / \pi d \mu \]

- orifice

\[ P_3 = (\rho Q_3 | Q_3 |) / 2 C_d A_0^2 \]
valve with variable area

\[ P_3 = \frac{(\rho |Q_3||Q_3|)}{2C_d^2(x)}A^2(x) \]

\[ Q_3 = C_d A_0 \left( \frac{1}{2}\right) \frac{P_3}{\rho} \right)^{1/2} \text{sgn} P_3 \]

**Fluid Inertia (Kinetic Energy)**

Long thin pipe creates a pressure drop associated with acceleration of a lump of fluid:
Derivation of fluid inertia for long pipe:

\[ P_3 = P_1 - P_2 \]

\[ p_{p_3} = \int P_3 \, dt - \int (P_1 - P_2) \, dt \]

\[ l_3 Q_3 = p_{p_3} \text{ or } Q_3 = \frac{p_{p_3}}{l_3} \]

**Sources**

Pump = pressure source (centrifugal); flow source (positive displacement)