particularly if there is a lack of agreement between the model predictions and experimental results.

REFERENCES


PROBLEMS

4-1 Make a bond graph model for each of the following electrical circuits. Use the inspection method whenever possible. If the circuit has voltage and current orientations, make the bond graph have equivalent power directions.

*Note:* The E-meter may be modeled as an open-circuit pair, and the auditor may be modeled as a resistance, $R_A$.

(a)  
(b)  
(c)  
(d)  
(e)  
(f)  
(g)  
(h)  
(i)  
(j)
4-2 Make a bond graph model of each of the electrical networks shown below.

4-3 Make a bond graph model of each of the translational mechanical systems shown below.

4-4 Find a bond graph model for the following mechanical networks.
4-5 For the following mechanical systems involving fixed-axis rotation, find a bond graph model in each case. In the rack-and-pinion system shown, several frictionless guides and bearings are omitted for clarity.

\[
\begin{align*}
V_3 : & \quad r \omega - u_1 \\
V_4 : & \quad -r \omega - u_1 \\
V_5 : & \quad -2 u_1 
\end{align*}
\]

4-6 Two hydraulic systems that are closely related but not identical are shown below. Make a bond graph model for each.

4-7 A simplified model of a water storage system is shown below. Assume the three tanks behave as (nonlinear) capacitances. Assume the three conduits have resistance only.
(a) Make a bond graph model.
(b) Introduce inertance effects for each of the conduits into your model for (a).

4-8 In the hydraulic system shown below, the pipes containing flows \( Q_4 \) and \( Q_5 \) have both inertia and friction effects present. The outflow line (\( Q_6 \)) has only friction effects. The control pump maintains a desired flow, \( Q_C \), independent of pressure. Make a bond graph model of the system.

4-9 In the positive-displacement pump shown below, the piston moves back and forth and the check valves act like unsymmetrical resistances, allowing relatively free forward flow while impeding the back flow greatly. Inertia effects are important in the outlet line, while the nozzle is a resistive restriction.

(a) Make a bond graph model that can be used to relate the mechanical port variables \( F, V \) to the fluid port variables \( P, Q \).
(b) Noting that an electrical diode is analogous to a fluid check valve, use the bond graph of part (a) to find an equivalent electrical network to the pump system.

4-10 Consider a permanent-magnet generator of the type often used to power bicycle lights:

(a) Suppose the generator to be an ideal gyrator in its transducer action. Make a bond graph model of the system.
(b) Suppose the lamp shortcircuits. Is your model from (a) still useful? If not, modify it so that it can handle this situation.

4-11 Two related hydraulic devices are shown below, a hydraulic jack and a ram:

(a) If the inertia of the sliding parts and friction at the seals are important, modify the models given in the figure for the devices to include these effects.
(b) If compliance of the working fluid for the jack and of the shaft for the ram is important, modify your models from (a) above to include these effects.

4-12 A schematic diagram of a basic d-c machine is shown below. The principal electrical effects are armature resistance and inductance and field resistance and inductance. The coupling between armature and shaft powers is modulated by the strength of the magnetic field, set by \( i_f \). A simple assumption for coupling is

\[
\tau = T(i_f)i_a = (K_i f)i_a
\]

and

\[
e_a = T(i_f)\omega = (K_i f)\omega
\]

where \( T(i_f) \) is the modulating value, assumed linear with \( i_f \).

(a) Make a bond graph model of the basic d-c machine using a modulated gyrator (MGY) as the heart of the coupling.

(b) Introduce mechanical rotational inertia and dissipation effects into your model from (a) above.

(c) Operate your model from (b) in the generator mode. What changes must be made?

(d) Calculate the power efficiency for the d-c machine operating as a motor and as a generator.

4-13 The network shown below includes a voltage-controlled current source. Make a bond graph model of the network using an active 2-port as the coupling element.

4-14 A load of mass \( M \) is suddenly dropped onto piston 1 from height \( L \).

(a) Make a bond graph model of the system, assuming piston 2 is locked in place by pins. Also assume the load stays joined to piston 1.

(b) If piston 2 is not locked in place, but pistons 1 and 2 were in equilibrium before the load was dropped, modify the model in (a).

4-15 Model the spring-loaded accumulator device shown below. Include inertia and dissipation effects.

4-16 A slab is shown below being quenched in a cooling bath. Assume the bath temperature is maintained constant at \( T_b \). You may neglect heat transfer from the ends and edges.

Skyhook $Q$ $Q$ $Q$ $T_b$
(a) Make a bond graph model of the cooling process.

(b) Suppose the water bath is in a tank of volume \( V_t \) and the temperature is not maintained constant. Modify the model to cover this case.

4-17 There is a simple analogy between lumped-parameter pseudo-bond graph models of conduction heat transfer systems and electrical circuits. Reinterpret the bond graph of Figure 3.18 as an electric circuit and point out the analogous thermal and electrical effort and flow variables.

4-18 For the device shown, construct a bond graph model. For the positive directions indicated, assign a power convention.

![Bond Graph Diagram](image)

4-19 The coupled mechanical and hydraulic system is used to isolate the mass, \( m \), from the ground motion, \( v(t) \). Construct a bond graph model, show your assumed positive \( e \) and \( f \) directions, and assign an appropriate power convention.

![Bond Graph Diagram](image)

4-20 A quarter car model of a vehicle is shown with an idealized actuator capable of generating any control force, \( F_c \).

(a) Construct a bond graph model, identify positive directions, and assign a power convention.

![Bond Graph Diagram](image)

(b) A voice coil actuator with winding resistance, \( R_w \), is to be installed as the force actuator to replace the idealized control force. The actuator will be voltage, \( e_c \), controlled. Include the force actuator in the quarter car model and construct the overall bond graph model.

![Bond Graph Diagram](image)

The hydraulic piston cylinder device is a fundamental component of many motion control systems. Hydraulics can generate large forces over large distances, which is necessary for big motion platforms such as flight simulators. The ideal form of the device shown is a transformer, as indicated in the bond graph.

An actual device has inertia, friction, leakage past the piston, and compliance of the volumes of oil on both sides of the piston. For the realistic effects shown, construct a bond graph model and assign a power convention.
4-22 Shock absorbers for automobiles are very sophisticated devices. The piston head has several levels of check valves to generate a force–velocity characteristic quite different from what might be expected from simply forcing oil through an orifice. Also, in order to provide volume for the piston rod as the shock is compressed, most shock absorbers are of a twin-tube construction, as shown below. The valve that connects the inner and outer tubes is called the foot valve.

(a) Construct a bond graph model incorporating the dynamic elements shown and assign a power convention.
(b) State your assumptions about more realistic dynamic effects and include them in your model.

4-23 Install the shock absorber model from Problem 4-22 into the quarter car model of Problem 4-20. Replace the damper with the shock model.

4-24 The system is similar to the example system of Figure 4.14. For this problem, the lever has mass, m, and cg moment of inertia, J_l. In addition, the pivot point has prescribed vertical motion set by the specified velocity, v_{12}(t). The spring at the left end has prescribed vertical velocity at the top, and there is a mass with spring and damper attached at the right end of the lever. The lever executes plane motion since it simultaneously translates and rotates.
(a) Derive the kinematic constraints for this system by relating v_1, v_2, and v_m to the cg velocity and angular velocity, v_g and \omega.
(b) State whether springs and dampers are positive in compression or tension and construct a bond graph model for this system.

4-25 Masses m_1 and m_2 move only horizontally. Mass m_1 has an input force prescribed, and m_2 has an attached spring, k. The two masses are connected by
a rolling element of mass, \(m_r\), cg moment of inertia, \(J_r\), and radius, \(R\). The rolling element rolls without slip at the contact point with each mass.

(a) Derive the kinematic constraints relating the mass velocities \(v_1\) and \(v_2\) to the velocity and angular velocity of the rolling element.

(b) Construct a bond graph model of this system and show an appropriate power convention.

4-26 A gantry robot is a large manufacturing machine that consists of a very stiff table-like structure, atop which sits a platform that can be driven in the \(X, Y\) plane. Suspended below the platform is the robot arm that will have a cutting tool of some kind attached at its lower end. The robot arm can be moved vertically, and the cutting tool can be moved in several directions. The result is a multi-axis cutting tool that can create large 3-D shapes. Although gantry robots are made as stiff and rigid as possible, sometimes small motions at the cutting tool end can limit the speed or precision of the machine.

Using the elementary bond graph shown, which neglects any resistance effects, the natural frequency of the oscillator is given by the formula

\[
\omega_n = (1/\mathbf{C})^{1/2} \text{ [rad/s]} \quad \text{or} \quad f_n = (1/\mathbf{C})^{1/2}/2\pi \text{ [cycles/s]} \text{ or [Hz].}
\]

(a) Using the formulas for acoustic inertia and capacitance given in the text, find an expression for \(\omega_n\) and \(f_n\) in terms of the dimensions of the device.

(b) What would be the natural frequency in air at standard conditions if \(d = 0.01\) m and \(D = l = L = 0.1\) m? Do you think you could hear this frequency if you blew across the end of the open tube as you would across an empty bottle?

4-28 Consider a length of hydraulic hose to be used to operate the ram of a high-speed actuator. There is a concern that the compliance in the system may slow down the system response. You are to decide whether the hydraulic compliance due to the compressibility of the hydraulic fluid or the flexibility of the hose walls is most important. Use these dimensions: nominal radius of the hose 15 mm, wall thickness 5 mm. Assume that the modulus of elasticity for the hose is about the same as for hard rubber and the bulk modulus for hydraulic oil has the value given in the Appendix.

4-29 Consider a system that is a combination of Figure 4.28 and Figure 4.29. That is, a hydraulic pump is connected to a hydraulic motor by one line to a relief valve and by another line from the valve to the motor. In the new system, both lines are long and to be represented by a combination of inertia, resistance, and capacitance, as in Figure 4.29. In addition, both the pump and the hydraulic motor are positive displacement machines, represented as in Figure 4.32. The hydraulic motor is driven by a d-c electrical motor as represented by the gyror in Figure 4.34.
The load that the hydraulic motor drives can be represented by a combination of a rotary mechanical inertia and a rotary resistance. Show a complete bond graph for the system after setting the atmospheric pressure to zero (using gage pressures).

4-30 The figure shows an accumulator similar to the one in Figure 4.26. In this case, there is a distinction between the fluid pressure $P_3$ and the pressure of the gas in the rubber bladder under some conditions. Before the fluid system is pressurized, when $P_3$ is zero, the pressure in the gas is $P_0$ and the volume of the gas is $V_0$. The bladder has a rigid section at the base that prevents it from expanding into the inlet pipe. Until the fluid pressure exceeds $P_0$, no fluid enters the accumulator; $Q_3 = 0$. However, after some fluid has entered the accumulator, the gas is compressed and the gas pressure and the fluid pressure are essentially identical.

Consider the volume of fluid that has entered the accumulator, $V_3 = \int_0^t Q_3 \, dt$, to be zero at $t = 0$ when the system is unpressurized. Make a sketch of the nonlinear capacitance relationship between $P_3$ and $V_3$ using the isentropic assumption discussed in the text. Note that the $P_3$ can rise to $P_0$ even when $V_0 = 0$ and that $P_3 \to \infty$ when $V_3 \to V_0$.

(a) Give an expression for the equivalent inertia (or equivalent mass) of the ram and pipe combination.
(b) Perform a sample calculation to find out what radius the pipe should have to make the fluid inertia of the pipe have the same effect on the effective mass of the system as the mass of the ram and load mass. Use the following parameters:

\[
\rho = 900[^{\text{kg}}/\text{m}^3], \quad l = 0.5[^{\text{m}}], \quad m = 40[^{\text{kg}}], \\
A = \pi R^2, \quad R = 50[^{\text{mm}}], \quad a = \pi r^2
\]

Answers:
(a) $I_{eq} = (m + \rho l A^2/a)$.
(b) $a = \rho l A^2/m, \quad r = 15[^{\text{mm}}]$.

4-31 The figure shows a pipe of area $a$ and length $l$, containing hydraulic fluid, connected to a hydraulic ram of area $A$. This combination will eventually be used to model a hydraulic system, and the concern is whether the pipe hydraulic inertia, $\rho l/a$, is significant in comparison with the mass, $m$, of the ram and its connected load mass.

The bond graph on the right shows the two inertia elements separately, and the one on the left shows the two inertias combined into a single equivalent inertia associated with the velocity of the ram, $V$. One way to compute the equivalent inertia is to express the kinetic energy $T = m V^2/2 + l Q^2/2$ (where $l = \rho l/a$) in terms of the mass velocity by recognizing that the mass velocity and the volume flow rate are related by the transformer law $Q = AV$.

(a) The voltage is a controlled effort source. Indicate this by including a signal or active bond with a double arrow impinging on an effort source.
(b) The d-c motor has inductance and resistance in the armature coils.
(c) Include a rotary inertia for the motor armature and the screw drive.
(d) Use a resistor to model the friction moment associated with the motor and the screw drive.
(e) A constant $S$ relates the angular velocity of the screw with the nut: $V_1 = S\omega$.
(f) The tube connecting the nut with the load mass is elastic with a spring constant $k$.
(g) The load mass has significant friction with the ground.

5

STATE-SPACE EQUATIONS AND AUTOMATED SIMULATION

In the last chapter, bond graph models were developed for many physical engineering systems. We first treated each energy domain separately, but our goal was always to connect the energy domains into overall system models using the same symbols and structure for the entire system. This was done in Section 4.4 of the previous chapter and will continue to be done for the rest of this text. The nine basic bond graph elements are capable of representing a very large cross section of engineering systems.

The reason for constructing bond graph models will be very apparent in this chapter. It will be demonstrated just how straightforward it is to derive system equations and to obtain computer solutions to these equations. In some cases we can go from a bond graph directly to a simulation using a computer program without first obtaining equations of motion manually. In other cases we will derive the equations of motion and use them to obtain analytical information about the system behavior. The real virtue of bond graph modeling becomes overwhelmingly apparent when one considers that, from a bond graph point of view, all systems appear the same. Thus, only one formulation and solution procedure is needed for any of the systems we model.

To enforce the virtue of having a formulation procedure that works for any bond graph, consider the physical system shown schematically in Figure 5.1a for part of an electric power steering system. A d-c motor with winding resistance, $R_w$, rotor inertia, $J_m$, and rotary damping, $b_r$, drives a flexible output shaft of rotary stiffness, $k_f$. The shaft is connected to a rack and pinion setup where the gear has radius, $R$, and the rack has mass, $m$. The rack is attached to a spring and damper, $k, b$. We desire to predict the motion of this system for a voltage input to the motor.

Without a procedure for deriving equations of motion, one might start by drawing a free body diagram of the entire system. This is done in Figure 5.1b. The electric circuit voltage, $\epsilon_{mf}$, is the back emf of the motor. The electrical torque, $\tau_e$, is the