Abstract - This paper proposes a technique to quantitatively and systematically search for decoupling among dynamic elements of a bond graph model, and partition models in which decoupling is found. The method can increase the efficiency and accuracy of simulation-based design by improving physical-domain model reduction and preventing the use of inappropriate decoupling assumptions.

A full model is first generated using the bond graph formalism. The relative contributions of the terms of the generalized Kirchoff loop and node equations are computed by calculating and comparing an aggregate measure of the power flow through the individual bonds connected to each 1- and 0-junction. Negligible aggregate bond power at a junction represents an unnecessary constraint term. Such bonds are replaced by a source modulated by the output of that junction. If separate bond graphs joined by modulating signals result, then the model can be partitioned into driving and driven subsystems. While the algorithm is not restricted to bond graph representations of system models, the formalism is shown to greatly facilitate its implementation and heighten the physical insight gained thereby.

The algorithm is demonstrated for a slider-crank mechanism. The case study illustrates that decoupling can be found without the modeler relying on a priori assumptions, and that the computation speed and ease of use of the model increase after partitioning. The validity of decoupling assumptions can be tracked as the design and environment change.

I. INTRODUCTION

The role of computer simulation in engineering design continues to increase as companies strive to gain competitive advantage by reducing the time required to move from concept to final product. As model complexity increases in step with advances in computer technology, the engineer remains well served to use “proper models” - simulation models with sufficient predictive ability but minimum complexity.

Proper Modeling [Wilson and Stein, 1995] has been defined as the systematic determination of the model of minimal complexity that a) satisfies the modeling objectives and b) retains physically meaningful variables for design. Techniques compatible with the proper modeling philosophy should be systematic and algorithmic, minimizing the need for a domain expert to override the algorithms and leverage his or her experience and intuition to generate the optimally reduced model. The techniques are to be applicable to hybrid models comprised of electrical, hydraulic, thermal, and multi-body mechanical components. To ensure that underlying assumptions remain valid throughout the process, the required complexity of the model should be reevaluated as the system parameters and environment change.

In addition to eliminating unnecessary complexity from a full model, the modeler may wish to systematically determine if boundaries or partitions exist in the model that allow application of model reduction techniques to two or more simpler submodels. A priori assumptions of one-way coupling within a system are often made to achieve the latter goal. As an example, consider a high-fidelity half-car model subject to small road undulations. For a suitably smooth road, the pitch motions will not affect the prediction of the longitudinal velocity and the vehicle can be approximated as a point mass. The longitudinal dynamic outputs may then be used to drive a pitch model that predicts the time response of the sprung mass rotation and suspension components [Louca et al., 2001]. This reduction becomes questionable when rougher road inputs excite pitch motions more vigorously. The intuition of the analyst may allow him or her to
recognize the extreme conditions that define the decoupled submodel extents; however, even the assumptions of experienced modelers may become violated as the system parameters or environment change during the design cycle. A quantitative measure to signal when underlying assumptions are on the verge of being violated is necessary.

If a model can be systematically decoupled into driving and driven partitions, then the energetic and power-conserving elements of the driven subsystem can be eliminated if they contain no outputs of interest. Such model reduction can facilitate computation of the governing equations of a system, and reduce the dimension of the search space during design optimization. If the modeling or design goal requires that the driven partition be retained, then the ability to work in two separate reduced-dimension design spaces is valuable. The ability to partition a system suggests the possibility of parallel simulation or separate partition design with simpler models. Hence it is the objective of this work to approach an arbitrary lumped-parameter model, quantitatively determine whether or not partitioning is possible, and demonstrate the benefits thereof.

The following section reviews the relevant state of the art in model reduction and partitioning. Section III proposes the method for quantifying coupling intensity and creating partition boundaries, leading to the algorithm in Section IV. Section V applies the algorithm to a slider crank mechanism. Discussion, summary, and conclusions follow.

II. BACKGROUND

Literature Review

The partitioning algorithm proposed herein leverages previous research on proper modeling in which energy or power-based metrics are used to gain insight into dominant system dynamics. Relevant to the current research are time-domain reduction techniques that can be applied to nonlinear systems. Rosenberg and Zhou (1988) investigated bond graph power flow as a tool by which model effects could be eliminated without introducing significant error. RMS power of each bond was calculated for a given input and time interval, and correlated with element importance. Ye and YouCEF-Toumi (1999) investigated the dynamic elements’ energy exchange pattern as a measure of their relevance. A sensitivity approach was used to estimate the contribution of neighboring bond energies to the bond energy associated with the output of interest. The goal of the work was to generate reduced models to predict an output of interest that may or may not be associated with high-energy dynamics.

Model partitioning has been studied in the optimal design community for concurrent design of large artifacts, with the goals of conceptual simplification of the system, generation of reduced-order subproblems, and parallel or distributed computation. Optimal model-based decomposition (OMBD) [MichelenA and Papalambros, 1995] has the end goal of dividing the all-at-once problem into equally weighted subproblems for easier optimization or utilization of parallel computing resources. OMBD partitions are mathematical optimal design constructs rather than physically meaningful partitions, and thus are not helpful for generating proper models.

Partial sensitivity derivatives have also been used in the design literature to indicate the strength of couplings among parts of a system, and have been proposed as a means to identify weak couplings within a mathematical model [Sobieszczanski-Sobieski, 1990]. The problem of partition identification based on minimal coupling was not addressed. Potentially costly symbolic or numerical evaluation of derivatives would be required for systems with large numbers of variables.

Formalism

The optimal language for proper modeling is one in which energy is conserved, power flow paths can be identified, and power flow associated with elements and their connections can be readily determined. Thus, bond graphs are the preferred formalism. 20SIM was selected as the bond graph user interface and simulation program for this work [20SIM, 2003].
III. METHODOLOGY

The power-conserving constraint equations represented by junction structure link the constitutive law variables of dynamic elements, and thus can give insight into the location of weak coupling upon:

- computation of an appropriate quantitative metric for the constraint terms
- comparison of the relative contribution of each term
- identification of negligible terms.

Partition identification, then, requires a search for groups of dynamic elements demarcated by negligible constraint terms. Here the insight provided by bond graphs is extremely valuable.

Examining the relative activity [Louca et al., 1997] of bonds at each 0- or 1-junction is proposed as a fundamental and meaningful way of unearthing negligible constraint equation terms. The junctions and their incident bonds represent all constraint equations in a model. Activity is chosen as the quantitative metric to compare the constraint term contributions. Activity $A$ of an element or bond over the time interval $[t_1, t_2]$ is defined as:

$$A = \int_{t_1}^{t_2} |P| dt$$

where $P$ is instantaneous power (product of effort and flow) of the element or bond. Activity is always positive, and either constant or monotonically increasing, over a given time interval.

**Proposition for constraint term assessment based on local junction structure activity comparison:**

**Article 1:** The activity of a bond attached to a 0- or 1-junction, compared to the activities of the other bonds at that junction, is a measure of the relative importance of the term represented by that bond compared to the other terms in the constraint equation represented by the junction. Low relative activity of bond $i$ at a junction implies that:

i) for a 0-junction with $n$ bonds, the flow $f_i$ can be neglected in the flow constraint equation

$$\sum_{j=1}^{n} f_j = 0$$

(2)

ii) for a 1-junction with $n$ bonds, the effort $e_i$ can be neglected in the effort equation

$$\sum_{j=1}^{n} e_j = 0.$$

(3)

**Article 2:** A power bond with low relative activity at a 0-junction (1-junction) can be converted to a modulated effort (flow) source. The modulating signal is the common effort (flow) from the junction. The effort (flow) is applied to the node at the other end of the bond, i.e., the end that is not adjacent to the 0-junction (1-junction) at which the bond is inactive. The half-arrow direction (direction of algebraically positive power flow) of the modulated source is the same as that of the original low-activity bond. Table 1, Scenario (i) illustrates the conversion.

**Article 3:** The activity of an “internal” junction structure bond (between two junction structure elements) must be compared to the activities of the other bonds at two separate elements, as shown in Scenario (i) of Table 1. If both Cases (a) and (b) apply, then the bond connecting the 0- and 1-junction is replaced by a bond with a source on either end, and the bond can be eliminated. If the local activity of an “external” junction structure bond (between the junction structure and an energetic element) is negligible, a trivial driven partition results as shown in Scenarios (ii) and (iii) of Table 1. The generalized impedance $Z$ (representing an $I$, $C$, or $R$ element) can then be eliminated from the model.

Note that in the table, the locally inactive bonds are causally weak. The issue of causal strength is addressed in the Discussion section.

Table 1 shows the internal bond connected to a 1- and a 0-junction. Other possible internal bond connections are to (M)TF or (M)GY elements. If both bonds are locally inactive compared to their respective junctions, then the transformer or gyrator can be eliminated. If one bond is locally inactive, a modulated source in sequence with an (M)TF or (M)GY results. The transformer or gyrator can be incorporated into the source, resulting in the equivalent conditioned junction structure shown in Table 2. The more compact representation will be used in the upcoming examples and case studies.
Article 4: Replacing a negligible constraint term with a modulated source amounts to locally replacing a two-way power flow path with a one-way signal. We use the modulating signal directions to indicate partitions according to the following definitions.

**Defn 5.1** The *conditioned* model or bond graph is the bond graph with all locally negligible power bonds converted to modulated sources.

**Defn 5.2** Given a conditioned bond graph, a *subgraph* \( S \) is a set of elements from that bond graph (including sources, energetic elements, and junction structure) that has no power bonds leading to any bond graph element outside the set. The subgraph may be connected to the rest of the bond graph by modulating signals, or unconnected.

**Defn 5.3** A *subgraph loop* \( S_L \) is a set of subgraphs in which each \( S \in S_L \) is connected to the previous by modulating signals directed into \( S \), and to the next by modulating signals directed out of \( S \). A subgraph loop is itself a subgraph. See Fig. 1.

**Defn 5.4** A *driving partition* is a subgraph \( S_D \) that is connected to at least one other subgraph strictly by modulating signals directed outwards from \( S_D \) to that subgraph. See Fig. 2. A subgraph within a subgraph loop can be a driving partition if it drives a subgraph outside that loop.

**Defn 5.5** A *driven partition* is a subgraph \( S_N \) that is not an element of a subgraph loop, that is connected to at least one other subgraph strictly by modulating signals directed inwards towards \( S_N \) from that subgraph (Fig. 2).

Internal modulating signals will generally exist within each subgraph. Note the difference in connectivity of the partition in Fig. 2 that is both driving and driven, and the connections among the subgraphs in Fig. 1, none of which qualify as driving or driven partitions. In Fig. 3 the three subgraphs in the loop collectively constitute a partition that is both driven and driving.

### Table 2. Transformer and gyrator conditioning.

<table>
<thead>
<tr>
<th>Transformer</th>
<th>Case A: ( M )</th>
<th>Case B: ( M )</th>
<th>Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gyrator</td>
<td>Case A: ( M )</td>
<td>Case B: ( M )</td>
<td>Equivalent</td>
</tr>
<tr>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

**IV. CONDITIONING AND PARTITION SEARCH ALGORITHM**

The local activity comparison and interpretation described in the previous section are incorporated into the preliminary algorithm shown schematically in Fig. 4 and Fig. 5. The first step is to construct a model that is taken to have at least adequate complexity, and that predicts the outputs of interest acceptably. The outputs of interest of the full model are recorded for later comparison with reduced or partitioned model outputs.

Local comparison entails dividing each bond activity by the maximum bond activity. Any activity ratio below a user-defined threshold is deemed negligible. In other words, eliminate terms \( i \) that satisfy

\[
\frac{A_i}{A_{\text{max}}} < \varepsilon.
\]

Upon choosing an initial threshold for local activity comparison and converting two-way power bonds to sources, the original outputs of interest and the conditioned model time histories are compared. Confirmation that the outputs of interest are predicted...
adequately implies that the constraint terms that were declared insignificant can indeed be eliminated. If partitions are then found, then agreement of the conditioned and full models is a sufficient condition for agreement of the partitioned and full models.

The modeler has the option to include external junction structure bonds as candidates for conversion, thus using the algorithm for simultaneous partitioning and elimination of dynamics. If external junction structure bonds are excluded, the algorithm will search strictly for partitions. Reduction techniques such as MORA can then be applied to each individual partition.

A possible bond conditioning exception exists when causally strong, locally inactive bonds connect the junction structure to constraining or parasitic elements. To create explicit dynamic equations when momenta are constrained (as in multibody systems), a causal effort must be imposed on 0-junctions representing constraint forces or Lagrange multipliers. Karnopp and Margolis [1979] propose the use of stiff, “parasitic” springs to approximate constraints. These elements are causally strong, and are intentionally tuned to not affect the actual system dynamics [Rideout and Stein, 2003]. van Dijk [1997] proposed the “controlled source” as a bond graph element to impose the constraint effort on the Lagrange multiplier 0-junctions. An effort source at the 0-junction is modulated by the constraint violation velocity, and the implicit integration scheme is used to generate the effort required to suppress the constraint violation velocity below the integration tolerance.

Parasitic and constraint elements, then, will have low local activity as they force relative velocity components of constrained bodies to nearly zero. Their retention is nonetheless desirable and they are not subject to bond conditioning and subsequent elimination.

V. CASE STUDY – SLIDER-CRANK MECHANISM

Fig. 6 is a schematic of a slider-crank mechanism with a translational spring connecting the massless slider to ground. The system parameters are as follows: \( a = 0.5, b = 5, J_1 = 0.125, J_2 = 0.17, m_2 = 0.5, k = 100, R_{spring} = 64, R_j = 20. \)

For this simple example absolute coordinates are used and the bond graph is based on the descriptor form of the equations of motion [Karnopp, 1997]. The full system bond graph is shown in Fig. 7. The bond graph that follows from the descriptor form has primitive coordinates for each energy storage element, explicit representation of Lagrange multipliers, and a modulated transformer for each term of the constraint equations.

An input torque is delivered to the crank so that the mechanism completes one revolution in 1.5 seconds. The activity values for this input are shown adjacent to each bond in Fig. 7. The modeling goal is to reduce the model as much as possible while still accurately predicting the
slider (and thus spring) displacement. A local activity threshold of 5% is selected, resulting in four inactive transformer bonds, the activities of which are circled in Fig. 7. For this case study only internal junction structure bonds are subject to conditioning, thus limiting the algorithm to a partition search rather than a dynamic element elimination tool. For example, the inertias \( J_1, m_s, \) and \( J_2 \) have locally inactive external junction structure bonds, but their retention will not inhibit the discovery of nontrivial partitions if any exist.

Consider the transformer with activity equal to 6, which is much less than the other activities at the “\( \lambda_1 \)” 0-junction. The transformer contributes the second term on the left-hand side of the constraint equation enforced by the 0-junction:

\[
a \sin \theta_1 \dot{\theta} + \frac{b}{2} \sin \theta_2 \dot{\theta}_2 + v_{bx} = \delta_1 = 0. \tag{5}
\]

As a result of the low activity, we reduce the equation to

\[
v_{bx} = -a \sin \theta_1 \dot{\theta}_1. \tag{6}
\]

The physical interpretation of this result is that the horizontal motion of the link center of gravity, and thus the slider and spring motion, is not affected significantly by the contribution of the link’s rotational velocity. The activity value of 6 is on the order of the activities from the other bonds at the 1-junction “\( \omega_2 \)”. The MTF thus cannot be completely eliminated, but can be replaced with a modulated effort source (MSe) that imposes an effort on the 1-junction based on an effort signal from the “\( \lambda_2 \)” 0-junction. Unlike a power bond, the effort signal imposes no flow quantity back on the 0-junction.

Similarly the bonds with activities 13 and 4 make negligible contributions to the effort equation enforced by the adjacent 1-junction, and the modulated transformers can be replaced with flow sources into 0-junctions “\( \lambda_3 \)” and “\( \lambda_4 \)”. The inertial forces/moments due to vertical motion of the link are thus not important effort inputs to the input-output bond graph path. The conditioned bond graph is shown in Fig. 8.

The spring element “C:k” is the bond graph element associated with the output of interest, and a single bond graph tree branch now connects the input torque source “MSe” to the spring. Bond graph loops have been removed by the substitution of signals for power bonds. The graph can be partitioned, with a “driving” partition consisting of the path from input to output, and a “driven” partition (containing the link dynamics and kinematics) that does not affect the response of the driving dynamics. Fig. 9 separates the partitions.

![Slider-crank bond graph with bond activities.](image)
Given that the output of interest is not in the driven partition, that partition can be completely eliminated. The crank velocity and spring state from the model thus reduced are plotted alongside the full model results in Fig. 10.

Table 3 shows that the number of integration steps and computation time decrease as a result of the model reduction.

As a result of applying a local power flow comparison metric to the junction structure of the bond graph, the number of primitive coordinates has been reduced. Two mass velocity primitive coordinates \(v_{bs} \) and \(\omega_j\) and the spring displacement \(d\) remain, with \(v_{bs}\) a function of \(\theta_1\), and \(d\) equal to \(-v_{bs}\). The spring state is predicted without the rotational degree of freedom of the link, which has been systematically eliminated.

Proc. IMAACA ’04, Bond Graph Techniques for Modeling Dynamic Systems, Genoa, Italy.
Fig. 10. Reduced model crank velocity and spring displacement

Table 3. Computation steps and time, MBDF integrator, tolerances $10^{-4}$

<table>
<thead>
<tr>
<th></th>
<th>Integration Steps</th>
<th>Computation Time [sec.]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Model</td>
<td>6484</td>
<td>1.49</td>
</tr>
<tr>
<td>Driving Partition</td>
<td>2228</td>
<td>0.78</td>
</tr>
</tbody>
</table>

VI. DISCUSSION

The decoupling search and partitioning algorithm has been shown to be systematic and quantitative, and gave results in the case study consistent with engineering intuition. The bond graph model formalism was specifically chosen to facilitate the proposed research, but other approaches are assumed suitable as long as the necessary power and energy variables can be extracted from the simulation environment. The algorithm is, however, limited to systems that can be modeled as a collection of discrete elements. Continuous systems described by partial differential equations can be subject to the algorithm if discretized or modeled as a collection of modes to which stiffness, damping, and compliance are assigned.

The algorithm development and example are presented in a scalar bond graph framework. Work is ongoing to determine the extent to which vector bond graphs must be decomposed during the junction structure activity search, conditioning, and partitioning.

The bond graph formalism allows easy visualization of causality issues. Bond graph causality is typically assigned to minimize dependent states or algebraic loops.

However, a numerically favorable causality assignment can be problematic if a causally strong input to a junction is locally inactive as shown in Fig. 11a.

Given that the modulating signal from a junction is its common power variable, it makes no sense to extract the effort from the 0-junction to modulate a flow source imposing flow on the 1-junction. If, however, causality is altered, then a modulated effort source can provide an input to the 1-junction as shown in Fig. 11b. Such a change in causality does not change the dynamics of, or connections within, the physical system being modeled.

Future work will increase the accuracy and scope of the algorithm by considering the contribution of modulating signals and improving the selection of the activity threshold. Modulating signals that exist prior to bond conversion, such as state modulating signals to coordinate transformations in multibody dynamics, can thwart partitioning. Figure 1 showed a subgraph loop for which partitions could be created if one of the modulating signals could be deemed unnecessary.

The selection of the threshold is somewhat arbitrary, as nonlinear systems in general will not allow an analytical mapping of the activity ratio at a partition boundary to the resulting discrepancy of the predicted system outputs before and after partitioning. Recent research on model quality assessment [Sendur et al., 2002] shows the potential to automate the selection of the activity threshold based on practical engineering design criteria.

Assuming a validated threshold that creates an acceptable partitioned model, the quantitative nature of the procedure allows the analyst to monitor the decoupling as the system parameters or environment change. Reducing the link length from 5 to 3, for example, and doubling the inertia parameters, results in the $MTF:acos\theta$, activity rising to 7.2% of the 1-junction maximum. Degradation of the output predictions can be observed, and the modeler is alerted to specific sites within the model where one-way coupling breaks down.
Intermediate models between the full and partitioned models are automatically suggested as newly significant power bonds are reinstated.

Quantitative model quality assessment also suggests a future approach for determining the range of inputs and parameters over which one-way coupling exists. Currently, the partitions are defined for a given design point, and the size of the region in design space in which the partitions are valid has not been formally treated.

VII. SUMMARY AND CONCLUSIONS

An algorithm has been proposed to systematically search a lumped parameter model, constructed using the bond graph formulation, for constraint equation terms that are negligible. Negligible terms are those whose activity (time integral of absolute value of power flow), when divided by the maximum constraint equation term activity, falls below a user-defined threshold. The terms are then eliminated by removing a causal input from the respective 0- or 1-junction by converting a power bond to a source modulated by the junction output power variable. In the event that collections of bond graph elements can be identified that are joined only by modulating signals in one direction, the model can be partitioned into driving and driven submodels. These reduced-order models can be simulated separately, and can be separately subjected to existing model reduction algorithms to identify the dominant dynamics of each partition. If the modeler is interested in a driving partition output, then the driven partition can be eliminated entirely. The modeler can validate a priori assumptions of decoupling, and track decoupling strength as design parameters and inputs change. The algorithm was demonstrated on a slider-crank mechanism in which the slider dynamics could be predicted without the rotational dynamics of the connecting link.

REFERENCES


