NAME: ANSWERS

STUDENT NUMBER: ______________

LAB DAY: ______________

ENGI 1040 – Electric Circuits
Winter 2015

Test 1
Sections 2, 4, and 6

Total Marks: 40

Only basic scientific (i.e., not programmable) calculators are permitted.

Clarity: Answer all of the following questions in the space provided. Marks will reflect the clarity of the presentation of your solution.

Units: State the units for all answers. Up to 1 mark will be deducted when an answer is presented without an appropriate unit.

Methods: When requested to use a specific method, this method must be used in order to obtain full marks for the question. If no specific method is requested, any appropriate method may be used.
**Question 1**  [8 MARKS - (a) 2 marks, (b) 2 marks, (c) 2 marks, (d) 2 marks]

Consider the following circuit. For time $t < 0$ seconds, both *Switch A* and *Switch B* are open. At $t = 0$ seconds, *Switch A* closes and then remains closed. At $t = 2$ seconds, *Switch B* closes and then remains closed.

(a) Sketch a graph of the current versus time for the current supplied by the voltage source, $i_s$, over the time interval from $t = 0$ to $t = 6$ seconds.

(b) Determine the total amount of charge that is supplied by the source during the interval from $t = 0$ to $t = 6$ seconds.

$$Q = i_s \times \Delta t = 5 \times 4 = 20 \text{ C}$$

(c) Determine the power supplied by the source during the interval from 0 to 2 seconds and from 2 to 6 seconds.

$$P = 20 \text{ V} \times 5 \text{ A} = 100 \text{ W}$$

$$P = 20 \text{ V} \times 10 \text{ A} = 200 \text{ W}$$

(d) Determine the energy consumed by resistor $R_1$ during the interval from 0 to 6 seconds.

$$E = 100 \text{ W} \times (6 - 2) \text{ s} = 400 \text{ J}$$
Question 2  [9 MARKS]

Consider the following circuit correctly setup by a student in the lab.

The current $i_s$ is measured to be 10 mA. The circuit is then accidently knocked onto the floor and rebuilt with one of the following fault conditions:

1. **R3 short**: 
   - resistor $R_3$ is shorted out (i.e., there is a short between point $a$ and point $b$)

2. **R3 open**: 
   - resistor $R_3$ is open circuited (i.e., one end of the resistor is not connected into the circuit)

3. **R1 open**: 
   - resistor $R_1$ is open circuited

4. **Current measurement error**: 
   - current $i_s$ is accidentally measured by putting the ammeter in parallel to $R_1$

You may assume that the ammeter is ideal and has an internal resistance of 0 $\Omega$.

For the reconstructed, faulty circuit, the student takes a new current reading for $i_s$ and gets a value of $i_s = 30$ mA. Complete the following table of expected current readings for $i_s$ for the four faults above. Identify the fault which has occurred.

<table>
<thead>
<tr>
<th>Fault</th>
<th>Expected Current Reading for $i_s$ (mA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) $R_3$ short</td>
<td>$i_s = \frac{15 \text{V}}{1 \text{k}\Omega} = 15 \text{mA}$</td>
</tr>
<tr>
<td>(2) $R_3$ open</td>
<td>$i_s = \frac{15 \text{V}}{1 \text{k}\Omega + 1 \text{k}\Omega} = 7.5 \text{mA}$</td>
</tr>
<tr>
<td>(3) $R_1$ open</td>
<td>$i_s = 0 \text{mA}$</td>
</tr>
<tr>
<td>(4) current measurement error</td>
<td>$i_s = \frac{15 \text{V}}{1 \text{k}\Omega + 1 \text{k}\Omega} = 30 \text{mA}$</td>
</tr>
</tbody>
</table>

The fault is current measurement error.
Question 3  [10 MARKS – (a) 5 marks, (b) 5 marks]

Consider the following circuit.

(a) What is the equivalent resistance, $R_{eq}$, of the entire circuit, as seen by the voltage source?

$$
R_{eq} = (3k\Omega \| 6k\Omega) \| [(2k\Omega + 3k\| 20k\Omega)]
$$

$$
= (2k\Omega) \| [(2k\Omega + 4k\Omega)]
$$

$$
= \frac{2 \times 6k\Omega}{2 + 6}
$$

$$
= 1.5k\Omega
$$

(b) Determine the current coming from the source, $i_s$. Then, use current divider twice, to determine first $i_1$ and subsequently $i_2$.

$$
i_s = \frac{15V}{1.5k\Omega} = 10mA
$$

$$
i_1 = i_s \cdot \frac{3k\Omega \| 6k\Omega}{(3k\Omega \| 6k\Omega) + (2k\Omega + 3k\| 20k\Omega)} \quad \text{(current divider)}
$$

$$
i_1 = 10mA \times \frac{2k\Omega}{2k\Omega + 6k\Omega}
$$

$$
i_1 = 2.5mA
$$

$$
i_2 = i_1 \cdot \frac{5k\Omega}{5k\Omega + 20k\Omega} \quad \text{(current divider)}
$$

$$
i_2 = 2.5mA \times \frac{5k\Omega}{5k\Omega + 20k\Omega}
$$

$$
i_2 = 0.5mA
$$
Question 4 \hspace{1cm} [6 MARKS – (a) 3 marks, (b) 3 marks]

Consider the circuit below.

\[+24\text{ V}\]

\[\begin{array}{c}
12\ \text{k}\Omega \\
\ \ \ \ \ \ \ \ \ \ \ \ \ a \\
\ \ \ \ \ \ \ \ \ \ \ \ b \\
12\ \text{k}\Omega \\
\ \ \ \ \ \ \ \ \ \ \ 1.5\ \text{k}\Omega \\
\end{array}\]

(a) Using voltage divider, calculate the voltage at node \(b\), relative to ground.

\[
V_b = 24 \times \frac{1.5\ \text{k}}{1.5\ \text{k} + 6\ \text{k} + 6\ \text{k}} \\
= 24 \times \frac{1.5\ \text{k}}{1.5\ \text{k} + 12\ \text{k}} \\
= 8\ \text{V} \\
V_b = 8\ \text{V}
\]

(b) Calculate the voltage of node \(a\) relative to node \(b\), \(V_{ab}\).

\[
V_a = 24 \times \frac{12\ \text{k}}{12\ \text{k} + 12\ \text{k}} \\
= 12\ \text{V} \\
V_{ab} = V_a - V_b \\
= 12 - 8 \\
= 4\ \text{V} \\
V_{ab} = 4\ \text{V}
\]
Question 5  [7 MARKS]

The variable resistance \( R \) in the circuit below is adjusted until \( i_1 = 10 \text{ mA} \) and \( i_2 = 40 \text{ mA} \). Determine \( R \) making use of KVL and KCL as appropriate.

Be sure to use the notation indicated on the circuit diagram when calculating appropriate circuit quantities. You may not need to calculate all the indicated voltages and currents.

\[ V_i = 10 \text{ mA} \times 5k = 50 \text{ V} \]

\[ V_i = 40 \text{ mA} \times 500 = 20 \text{ V} \]

\[ V_i + V_j = V_L \Rightarrow V_L = 20 - 50 = -30 \text{ V} \]

\[ E = V_i + V_L = V_i = 80 - 20 - 60 \text{ V} \]

\[ i_j = \frac{60}{3k} = -10 \text{ mA} \]

\[ i_R = i_i + i_j = 40 - 10 = 30 \text{ mA} \]

So \( R = \frac{V_L}{i_R} = \frac{60}{30} = 2k \)

\[ R = 2k \Omega \]