NAME: SOLUTIONS

STUDENT NUMBER: 

LAB DAY: 

ENGI 1040 – Electric Circuits
Winter 2015

Test 2
Sections 1, 3, and 5

Total Marks: 40

Only basic scientific (i.e., not programmable) calculators are permitted.

Clarity: Answer all of the following questions in the space provided. Marks will reflect the clarity of the presentation of your solution.

Units: State the units for all answers. Up to 1 mark will be deducted when an answer is presented without an appropriate unit.

Methods: When requested to use a specific method, this method must be used in order to obtain full marks for the question. If no specific method is requested, any appropriate method may be used.
Question 1  [12 MARKS - 3 marks per part]

(a) A resistive load of 20 Ω has 8×10^19 electrons travel through it over a 500 ms interval. Calculate (i) the current I passing through the load, (ii) the potential difference V across the load, and (iii) the energy W dissipated by the load during the 500 ms interval.

Recall that 6.25×10^18 electrons is equivalent to 1 C of charge.

\[ Q = 6 \times 10^{19} / 6.25 \times 10^{18} = 12 \text{ C} \]

\[ I = Q / t = 12 \times 8 \text{ C} = 25.6 \text{ A} \]

\[ V = IR = 25.6 \times 20 = 512 \text{ V} \]

\[ W = P \times t = 512 \times 25.6 \times 0.5 \]

\[ W = 6553.6 \text{ J} \]

(b) How much current is drawn by a 400 W electric heater that is plugged into a 120 V(emu) household circuit? If the circuit is protected by a 15 A(fus) fuse, what is the largest number of 400 W heaters that can be plugged into the circuit without the fuse blowing? Be sure to show your reasoning.

\[ P = V \times I \quad \Rightarrow \quad I = P / V = 400 \text{ W} / 120 \text{ V} = 3.33 \text{ A} \]

\[ \# \text{ heaters} \leq \frac{15 \text{ A}}{3.33 \text{ A}} = 4.5 \]

Answer: \(\leq 4 \text{ heaters}\)

\[ \text{heater current} = 3.33 \text{ A} \]

\[ \text{number of heaters} = 4 \]
(c) A 9V battery is connected across a 10 \( \Omega \) resistor. An ammeter with an internal resistance of 0.1 \( \Omega \) is used to measure the current through the resistor. What is the voltage across the resistor with the ammeter in the circuit?

\[
V_R = \frac{9}{10 + 0.1} = 0.91 \text{ V}
\]

**voltage across resistor = 0.91 V**

(d) Determine the equivalent resistance looking in terminals \( a-b \) for each of the following circuits which contain multiple resistors of value \( R \) ohms.

![Diagram](image)

\[ R_{ab} = R \]

\[ R_{ab} = R \]

\[ R_{eq} = \frac{R}{2} + R \]

\[ R_{ab} = \frac{1}{3} R \]
Question 2  [6 MARKS]

Consider the design of the voltage divider circuit shown below. The following are the requirements for the system:

1. The output voltage is to be $v_2 = 10$ V.
2. The resistors must be selected such that they do not dissipate more than $\frac{1}{4}$ W.
3. The smallest resistors possible to satisfy requirements (1) and (2) should be selected.

Determine $R_1$ and $R_2$ of the circuit.

\[ V_L = 15 = 15 \frac{R_L}{R_1 + R_2} \]
\[ \Rightarrow R_2 = 2R_1 \quad \text{(1)} \]

Since $R_2$ is larger than $R_1$ and same current flows through both $P_2 > P_1$ (since $P = I^2R$)

\[ P_1 \leq \frac{1}{4}\text{ W and } P_2 \leq \frac{1}{4}\text{ W} \]

so \[ \frac{V_1^2}{R_1} \leq \frac{1}{4} \Rightarrow \frac{10^2}{R_1} \leq \frac{1}{4} \]

\[ R_1 \geq 400 \Omega \]  \hspace{2cm} \text{from (1) above, } R_1 \geq 200 \Omega.

Another solution, which assumes $P_1 + P_2 \leq \frac{1}{4}$

\[ \Rightarrow \frac{15^2}{R_1 + R_2} \leq \frac{1}{4} \]

\[ R_1 + R_2 \geq 900 \Omega \]

From (1), $R_2 \geq 900 \Omega$

\[ R_1 \geq 700 \Omega \]

and $R_2 \geq 600 \Omega$.  

\[ R_1 = 200 \Omega \]

\[ R_2 = 400 \Omega \]
Question 3 [13 MARKS (a) 5 marks, (b) 4 marks, (c) 4 marks]

Consider the circuit below.

(a) Determine the equivalent resistance seen by the voltage source.

\[ R_{eq} = 18 \parallel \left[ \left( 1.5 \parallel \left( 2 \parallel 5 \right) \right) + 5 + 7 \right] \]
\[ = 18 \parallel \left[ \left( 1.5 \parallel 7 \right) + 5 + 7 \right] \]
\[ = 18 \parallel \left[ 1 + 5 + 7 \right] \]
\[ = 18 \parallel 9 \]
\[ = 6 \]

(b) Determine (i) the current delivered by the voltage source, \( i_S \), (ii) the power delivered by the source, \( P_S \), and (iii) the fraction of the power delivered by the source that is dissipated by the 18 \( \Omega \) resistor.

\[ i_S = \frac{12}{6} = 2 \, A \]
\[ P_S = V_S \times i_S = 12 \times 2 = 24 \, W \]
\[ P_{18\Omega} = \frac{12^2}{18} = 8 \, W \]
\[ \frac{P_{18\Omega}}{P_S} = 24 = \frac{1}{\frac{3}{8}} \]

(c) Using current divider, determine the currents \( i_0 \) and \( i_1 \).

\[ i_0 = i_1 \times \frac{18}{18 + 9} = \frac{2}{3} \times 1.33 \, A \]
\[ i_1 = i_0 \times \frac{1.5}{1.5 + 7} = \frac{4}{9} \times 1.33 \, A \]

\[ i_0 = 1.33 \, A \]
\[ i_1 = 0.44 \, A \]
Question 4  [9 MARKS - (a) 4 marks, (b) 5 marks]

(a) Consider the circuit below. The voltage source, $v_s$, can be adjusted from 0 V to 50 V, resulting in different voltages for $v_{ab}$.

![Circuit Diagram](image)

(i) Making use of voltage divider, write an expression for $v_{ab}$ as a function of $v_s$.

\[
\begin{align*}
v_a &= \frac{3k}{1 + 3k} v_s = \frac{v_s}{3} \\
v_b &= \frac{6k}{7k + 6k} v_s = \frac{2v_s}{3} \\
v_{ab} &= v_a - v_b = -\frac{v_s}{3}
\end{align*}
\]

So, $v_{ab} = -\frac{v_s}{3}$

(ii) If the voltage source is adjusted so that $v_{ab} = -8$ V, what is $v_s$?

\[-8 = -\frac{v_s}{3} \Rightarrow v_s = 24 \text{ V}\]

(b) The circuit now has a 6 kΩ ohm resistor connected between points $a$ and $b$ and the voltage source is adjusted so that $v_s = 30$ V. If $i_3 = 4$ mA, what is the value of $v_{ab}$?

Be sure to use the notation indicated on the circuit diagram when calculating appropriate circuit quantities. You may not need to calculate all the indicated voltages and currents.

\[
\begin{align*}
v_7 &= i_7 v_7 = 4 \times 7k = 28 \text{ V} \\
v_3 &= v_1 + v_2 \quad \text{(KCL)} \\
v &= 70 - 12 = 58 \text{ V} \\
i_1 &= \frac{v}{6k} = 3 \text{ mA} \\
i &= i_1 + i_5 \quad \text{(KCL)} \\
i_5 &= 3m - 4m = -1m \\
v_{ab} &= i_5 \times 6k = -1m \times 6k \\
v_{ab} &= -6 \text{ V}
\end{align*}
\]

So, $v_{ab} = -6 \text{ V}$