Thick Shells – Shrink Fits Problem set

1. The cylinder of a hydraulic ram is 254 mm internal diameter and 50 mm thick. Find the maximum hoop stress in the material and the radial and hoop stress at a point in the barrel 19 mm from the inner surface when the fluid pressure inside the cylinder is 9.24 MPa.

   (Ans: 28.83, 4.6, 24.2 MPa)

2. Determine the radial and hoop stresses at all points in a thick-walled cylinder of internal and external radii 76 mm and 152 mm respectively, when acted upon by internal and external Pressures of 124 MPa and 46 MPa respectively. Show by means of a diagram how each stress varies over the thickness. For your distribution you may calculate the hoop and radial stresses at radii 76, 100, 125 and 152 mm.

   ANS:
   \[
   \begin{array}{c|c|c}
   r & 76 & 152 \\
   \sigma_r & -124 & -46 \\
   \sigma_t & 84 & 6 \\
   \end{array}
   \]

3. A compound tube is made by shrinking one tube on another, the final dimensions being:
   - External diameter - 254 mm
   - Internal diameter - 150 mm
   - Diameter at junction of tubes - 228 mm

   If the radial pressure on the external surface of the inner tube at the common 114 mm radius is 27.58 MPa, determine the maximum tensile stress in the material of the outer tube.

   (Ans: 257 MPa)

4. A mild steel sleeve 14 cm inside diameter and 20 cm outside diameter is subjected to an internal pressure of 45 MPa. The sleeve is rigidly hooped so that the change in external diameter is zero. Determine the values of the radial and circumferential stresses at the inner and outer surfaces. Proceed from Lame’s equations and neglect axial stresses. \( \nu = 0.3 \)

   (Ans: inner: -45 MPa, +2.2 MPa
   outer: -32.8 MPa, -9.9 MPa)

5. Show that the maximum circumferential stress in a thick walled cylinder subjected to a uniformly distributed external pressure, the internal pressure being zero, is compressive and occurs at the bore. A thick cylinder 20 cm external and 10 cm internal diameter is subjected to a uniform pressure on its external surface only. Under the condition the change in bore volume is not to exceed 0.1% of the original. Obtain the value of the maximum circumferential stress in the cylinder and the permissible external pressure assuming the axial stress to be zero. \( E = 207 \ \text{GPa}, \ \nu = 0.3 \) (Ans: -121.6 MPa; 45.6 MPa)
6 A bronze cylinder of 0.3 m internal diameter and 0.4 m external diameter is surrounded by a closely fitting steel sleeve of 0.45 m external diameter. Calculate the maximum hoop stresses in the (sleeve) steel and (cylinder) bronze when an internal pressure of 30 MN/m² is applied to the compound cylinder, assuming that before the application of this pressure, the contact stress at the common surface is zero.

\[ E_{\text{steel}} = 200 \text{ GN/m}^2 \text{ and } \nu = 0.28 \]
\[ E_{\text{bronze}} = 100 \text{ GN/m}^2 \text{ and } \nu = 0.35 \]

(Ans: \( p = 10.13 \text{ MPa} \), \( \sigma_c = +60.8 \text{ MPa at inner surface of inner cylinder} \), \( \sigma_c = +86.4 \text{ MPa at inner surface of outer cylinder} \))

7 A compound steel cylinder has a bore of 80 mm and an outside diameter of 160 mm and at the common surface a diameter of 120 mm. Find the radial pressure at the common surface which must be provided by shrinkage, if the resultant maximum hoop tension in the inner cylinder under a superimposed internal pressure of 60 MPa is to be half the value of the maximum hoop tension which would be produced in the inner cylinder if that cylinder alone were subjected to an internal pressure of 60 MPa.

Determine the final hoop tensions at the inner and outer surfaces of both cylinders under the internal pressure of 60 MPa, and sketch a graph to show how the hoop tension varies across the cylinder wall.

(Ans: 6.11 MPa, 77.42 Mpa, 55.7 Mpa, 78 Mpa, 39.66 MPa)

8 A steel cylinder is 160 mm ID and 320 mm OD. If it is subject to an internal pressure of 150 MPa, determine the radial and tangential stress distributions and show the results on a plot (using a spreadsheet). Determine the maximum shear stress in the cylinder. Assume it has closed ends.

\( \sigma_r = 250 \text{ to } 100 \text{ MPa}, \sigma_t = 0 \text{ to } \pm150 \text{ MPa}, \tau_{\text{max}} = 200 \text{ MPa}. \)

9 A cylinder is 150 mm ID and 450 mm OD. The internal pressure is 160 MPa and the external pressure is 80 MPa. Find the maximum radial and tangential stresses and the maximum shear stress. The ends are closed.

\( \sigma_r = 20 \text{ to } \pm60 \text{ MPa, } \sigma_t = \pm80 \text{ to } \pm160 \text{ MPa, } \tau_{\text{max}} = 90 \text{ MPa}. \)

10 A cylinder has an ID of 100 mm and an internal pressure of 50MPa. Find the needed wall thickness if the factor of safety n is 2.0 and the yield stress is 250 MPa. Use the maximum shear stress theory, i.e. maximum shear stress = yield strength/2n

( wall = 61.8 mm thick )

11 A 400 mm OD steel cylinder with a nominal ID of 240 mm is shrunk onto another steel cylinder of 240 mm OD and 140 mm ID. The radial interference \( \delta \) is 0.3 mm. Use Young's Modulus \( E = 200 \text{ GPa} \) and Poisson's Ratio \( \nu = 0.3 \). Find the interface pressure \( p_i \) and plot the radial and tangential stresses in both cylinders. Then find the maximum internal pressure which may be applied to the assembly if the maximum tangential stress in the inside cylinder is to be no more than 140 MPa.

\( p_i = \pm120.3 \text{ MPa}. \) :: inner cylinder : \( \sigma_r = \pm365 \text{ to } \pm244 \text{ MPa, } \sigma_t = 0 \text{ to } \pm120.3 \text{ MPa.} \) :: outer cylinder : \( \sigma_r = 256 \text{ to } 135 \text{ MPa, } \sigma_t = \pm120.3 \text{ to } 0 \text{ MPa.} \) :: maximum internal pressure = 395 MPa.)
A cylinder with closed ends has outer diameter \( D \) and a wall thickness \( t = 0.1D \). Determine the percentage error involved in using thin wall cylinder theory to calculate the maximum value of tangential stress and the maximum shear stress in the cylinder.

(tangential stress \( \pm 9.75\% \); max. shear stress \( \pm 11.1\% \))

An experimental vessel for high pressure chemical experiments is cylindrical 7 in. OD by 18 in. Long. There is a central bore 2.5 in. diameter by 12 in. long with the ends closed by 3 1/2 in. Threaded plugs tightened against seals. The pressure will be 75,000 psi. (A) Determine the principal and the shear stresses at ID and OD of the central part of the vessel. (B) Explain why material selection based upon the shear strength is particularly important? For a factor of safety of 1.25 what yield strength \( S_y \) is required?

The cylinder of a slow-speed reciprocating-piston boiler-feed pump has an inside diameter of 200 mm and operates against a pressure of 9.25 MPa. Cast iron with a tensile strength of 175 MPa is used. For a safety factor of 4.0 what should be the minimum wall thickness?

A naval gun with a bore of 100 mm must sustain a maximum pressure of 200 MPa. What must be yield strength of the material if weigh limits the outside diameter to 300 mm at breech end, the maximum shear stress theory of failure is used and the factor of safety is 1.3 ?

A rule of thumb for the outside diameter of a cat-iron hub is to make it twice the shaft diameter. If the shaft is steel and solid, and the stiffening effect of spokes can be neglected, determine the maximum tensile stress in the hub corresponding to an interference of 0.001 inch per inch of shaft diameter.

\[ E_{ci} = 15 \times 10^6 \text{ psi}, \quad E_{st} = 30 \times 10^6 \text{ psi}, \quad \square_{ci} = 0.25 \text{ and } \square_{st} = 0.3 \]

Forged steel tires may be shrunk fitted onto cast-steel locomotive driving wheels and replaced when worn. An initial interference of about 1/64 in. per foot of fit diameter has been used. The interference is removed by heating the tire, followed by assembly, the coefficient of expansion for steel is \( 6.3 \times 10^{-6} /\degree \text{F} \). A tire for an electric locomotive has 42 in. Outside diameter, is 1 1/2 in. thick, 4 in. wide and flanged. Determine the temperature to which it must be heated for assembly and the stress and pressure on the tire after it cools.

A hardened steel pinion with a yield strength of 690 MPa, a bore of 75 mm, an equivalent outside diameter of 150 mm must transmit a torque of 2100 N.m after being shrunk fitted to a steel shaft. The minimum expected coefficient of friction is 0.10, but use a factor of safety of 3.0 (a) what interference is needed? To what temperature should pinion be heated: Expansion coefficient \( \square \) is \( 11.5 \times 10^{-6}/\degree \text{C} \) (b) is the resulting stress reasonable?

A steel cylinder having an internal diameter of 150 mm and an external diameter of 200 mm is shrunk over a tube having an internal diameter of 100 mm. If the compound cylinder is designed so that maximum tensile stress in the tube will be equal to the maximum tensile
stress in the jacket after an internal pressure of 165 MPa is applied, determine the initial 
shrinkage pressure between the tube and the jacket and the interference required to produce 
this shrinkage pressure

20 A thick-walled cylinder with an inside diameter of 150 mm was designed with a minimum 
wall thickness to withstand an internal pressure of 105 MPa and to develop a maximum 
tensile stress of 175 MPa. If the internal pressure must be doubled what is the minimum 
thickness for a shrunk-fitted jacket that will permit the maximum tensile stress in the 
assembly from exceeding 175 MPa?

21 A bronze tube, 60 mm external diameter and 50 mm bore, fits closely inside a steel tube of 
external diameter 100 mm. When assembly is at uniform temperature of 15°C the bronze 
tube is a sliding fit inside the steel tube, that is, the two tubes are free from stress. The 
assembly is now heated uniformly to a temperature of 115°C. Calculate the radial shrinkage 
pressure between the mating surfaces and the thermal circumferential stresses induced at the 
inside and outside surfaces of each tube.

\[ E_{st} = 200 \text{ GPa}, \quad \nu_{st} = 0.3 \quad \alpha_{st} = 12 \times 10^{-6/\circ C} \]

\[ E_{br} = 100 \text{ GPa}, \quad \nu_{br} = 0.33 \quad \alpha_{br} = 19 \times 10^{-6/\circ C} \]