§ 3 Minimization Techniques and Digital System Design

§ 3.1 Combinational Circuits Minimization Techniques

1. Much of the simplifying work was done to the datapath
   a. Boolean Logic Theorems
   b. K-Map (3-6 variables, more than that will be less useful)
   c. Tabular Method (Based on Quine-McClusky method)
   d. Use Software (Like Espresso)

2. Example:
   \[ F(A, B, C, D) = \sum (4, 5, 6, 8, 9, 10, 13) \]
   \[ d(A, B, C, D) = \sum (0, 7, 15) \]
   a. Arrange the minterms based on the hamming weight, that is, the number of 1’s in the minterm
   b. Arrange in the table
c. Find the essential groups, that is, group cover most variables

§ 3.2 Sequential Logic Circuits Minimization

<table>
<thead>
<tr>
<th>Present State</th>
<th>Next State</th>
<th>Output (y₁, y₀)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Input x = 0</td>
<td>Input x = 1</td>
</tr>
<tr>
<td></td>
<td>Input x = 0</td>
<td>Input x = 1</td>
</tr>
<tr>
<td>M</td>
<td>m</td>
<td>n</td>
</tr>
<tr>
<td>N</td>
<td>q</td>
<td>n</td>
</tr>
<tr>
<td>O</td>
<td>o</td>
<td>n</td>
</tr>
<tr>
<td>P</td>
<td>p</td>
<td>r</td>
</tr>
<tr>
<td>Q</td>
<td>s</td>
<td>r</td>
</tr>
<tr>
<td>R</td>
<td>q</td>
<td>r</td>
</tr>
<tr>
<td>S</td>
<td>o</td>
<td>r</td>
</tr>
</tbody>
</table>
§ 3.2  Digital System Design Example: Serial to Parallel Converter (STOP)

1. Specification

- Synchronous reset (R)
- Input A is asserted for exactly one clock period prior to the arrival of serial data on input D
- For the next four clock cycles, data arrives serially on D
- Device collects four bits of serial data and output them in parallel at output Z
- When parallel data appears, signal DONE asserts
- Z and DONE must remain asserted for one full clock cycle
- During the clock cycle when the parallel data is at Z, the device may receive another pulse on A, indicating that new data will arrive
  - Yes, be prepared to receive
  - No, goes to reset state after sending out Z

2. Timing diagram

3. Step 1: Moore / Mealy Machine

⇒ Functionally, it is always possible to build any specification as wither a Moore machines or a Mealy machine.
⇒ Primary difference is in the timing of the outputs.
4. Three practical effects to consider:
   ➔ In Moore machine, outputs settle to their final values a few gate delays after the active clock edge. They are constant for the remainder of the clock period, even if inputs happen to change during the clock period.
   ➔ It isolates outputs from inputs.
   ➔ Mealy machine respond one clock period earlier than Moore machine to input changes, but also allows outputs to follow spurious input changes.
   ➔ Noise on the input lines may be transferred to the outputs.
   ➔ Moore machine may require more states than that from a Mealy machine.

   ➔ For the design example, output must be present during the clock cycle following the last input. Outputs required to be constant during the entire period, a Mealy machine can’t be used.

5. Step 2: construct of state table: follow a structured methodology
   ➔ Two approaches:
     (1) State diagrams
        **Advantage**: simple, easy to start with
        **Disadvantage**: limit to relatively small devices, but enough for STOP
        ➔ Start with a state that is easily described in words. If there’s a reset state, it is always good place to start with. Write a complete word description of each state as it’s described.

        ➔ **Reset State (S0)**
        Entered at the end of any clock where R = 1
        Stays until A = 1
        DONE = 0, output Z is unspecified.
        
        Then decide what to do in S0 for various conditions on inputs.
→ **State S1**

Entered from S0 when R = 0 and A = 1
Data on D must be saved at the clock edge for later output.
DONE = 0, output Z unspecified.

... ... ...
Repeat until the complete state diagram is achieved.

### (2) Transition List Approach

**Advantage:** good for problems that are too complex for a state diagram to be constructed.

<table>
<thead>
<tr>
<th>Present State</th>
<th>Condition</th>
<th>Next State</th>
<th>Data Transferred</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>S0</td>
<td>R + (A)</td>
<td>S0</td>
<td>None</td>
<td>0–</td>
</tr>
<tr>
<td>S0</td>
<td>(R).A</td>
<td>S1</td>
<td>None</td>
<td>0–</td>
</tr>
<tr>
<td>S1</td>
<td>(R)</td>
<td>S2</td>
<td>Store Bit1</td>
<td>0–</td>
</tr>
<tr>
<td>S1</td>
<td>R</td>
<td>S0</td>
<td>None</td>
<td>0–</td>
</tr>
<tr>
<td>S2</td>
<td>(R)</td>
<td>S3</td>
<td>Store Bit2</td>
<td>0–</td>
</tr>
<tr>
<td>S2</td>
<td>R</td>
<td>S0</td>
<td>None</td>
<td>0–</td>
</tr>
<tr>
<td>S3</td>
<td>(R)</td>
<td>S4</td>
<td>Store Bit3</td>
<td>0–</td>
</tr>
<tr>
<td>S3</td>
<td>R</td>
<td>S0</td>
<td>None</td>
<td>0–</td>
</tr>
<tr>
<td>S4</td>
<td>(R)</td>
<td>S5</td>
<td>Store Bit4</td>
<td>0–</td>
</tr>
<tr>
<td>S4</td>
<td>R</td>
<td>S0</td>
<td>None</td>
<td>0–</td>
</tr>
<tr>
<td>S5</td>
<td>(R).A</td>
<td>S1</td>
<td>None</td>
<td>0–</td>
</tr>
<tr>
<td>S5</td>
<td>R + (A)</td>
<td>S0</td>
<td>None</td>
<td>0–</td>
</tr>
</tbody>
</table>
Principle of mutual exclusion

- Used to aid the design process and check state diagrams for errors.
- Logic expressions on arcs leaving any node must be pair-wise mutually exclusive, that is, no two expressions on different arcs leaving the same node can be true simultaneously. E.g., \((R).A.[R + (A)] = 0\)

Diagram for circuit design
6. Step 3: VHDL coding

```vhdl
ENTITY stop IS
    PORT (R, A, D, CLK : IN BIT;
          Z : OUT BIT_VECTOR (3 DOWNTO 0);
          DONE : OUT BIT);
END stop;

ARCHITECTURE fsm_rtl OF stop IS
    TYPE state_type IS (S 0, S1, S2, S3, S4, S5);
    SIGNAL state: state_type;
    SIGNAL shift_reg : BIT_VECTOR (3 DOWNTO 0);
BEGIN
    State_decoding: PROCESS (CLK)
    BEGIN
        IF (CLK = '0') THEN
            CASE state IS
            WHEN S 0 =>
                IF (R='1' OR A='0') THEN
                    state <= S0;
                ELSIF (R='0' OR A='1') THEN
                    state <= S1;
                END IF;
            WHEN S 1 =>
                shift_reg <= D & shift_reg (3 DOWNTO 1);
                IF (R='0') THEN
                    state <= S2;
                ELSIF (R='1') THEN
                    state <= S0;
                END IF;
            WHEN S 2 =>
                shift_reg <= D & shift_reg (3 DOWNTO 1);
                IF (R='0') THEN
                    state <= S3;
                ELSIF (R='1') THEN
                    state <= S0;
                END IF;
            END CASE;
        END IF;
    END PROCESS;
END fsm_rtl;
```

WHEN S3 =>
    shift_reg <= D & shift_reg (3 DOWNTO 1);
    IF (R='0') THEN
        state <= S4;
    ELSIF (R='1') THEN
        state <= S0;
    END IF;
WHEN S4 =>
    shift_reg <= D & shift_reg (3 DOWNTO 1);
    IF (R='0') THEN
        state <= S5;
    ELSIF (R='1') THEN
        state <= S0;
    END IF;
WHEN S5 =>
    IF (R='0' AND A='1') THEN
        state <= S1;
    ELSIF (R='1' OR A='0') THEN
        state <= S0;
    END IF;
    END CASE;
END IF;
END PROCESS state_decoding;

Output_proc: PROCESS (state)
BEGIN
    CASE state IS
        WHEN S0 TO S4 =>
            DONE <= '0';
        WHEN S5 =>
            DONE <= '1';
            Z <= shift_reg;
    END CASE;
END PROCESS output_proc;
END fsm_rtl;
§ 3.3 Implement a State Machine

Questions 1: Mealy machine or Moore machine?
Questions 2: Is this state diagram complete?
Questions 3: The timing diagram?
Questions 4: Realize with the fewest positive edge triggered T-FF?
Questions 5: What if D-FF will be used, or J-K FF? What if one hot design?
§ 3.4 Design Example (Microwave Oven Controller)
§ 3.5 ASIC Design Methodology

1. Generally speaking, Top-Down Design Process followed by Bottom-up Implementation

2. During the top-down design process, top level entity is recursively divided by using the divide-and-conquer strategy until all leaf components of the design tree become manageable.

3. Factors to be considered between a semi-custom ASIC device (FPGA, PLD) and a full-custom ASIC device (CMOS IC).
   a. Speed
   b. Number of input and output pins
   c. Size of the circuit
      i. Enough FFs / Latches / Gates
      ii. Then actual mapping
   d. If constraints from CLB and IOB and interconnections between blocks are not prohibitive
      ➔ ASIC might be good option if available
   e. Number of gates < 1M
   f. Number of pins < a few hundreds ➔ available for FPGA
   g. Volume of product
      i. < hundreds of thousands ➔ semi-custom
      ii. > millions of devices ➔ full-custom less expensive
      iii. Prudent to first bring out an FPGA (less development time), in the mean time full-custom ASIC
   h. If involves analog ➔ full custom normally required

4. ASIC Design flow recommended by CMC
Synopsys