Electrical and Computer Engineering
Memorial University of Newfoundland

ENGI 9876 - Advanced Data Networks
Fall 2018

Problem Set 2

(Due: Friday, October 19, 2018)

Question 1

Consider an M/M/1 model of a queue in a packet switched network, for which $\lambda = 50$ packets/s and $\mu = 60$ packets/s.

(a) Determine the probability that between 5 and 7, inclusive, packets arrive at the queue in a 100 ms period.

(b) Determine the probability that a packet is longer than 5000 bits, given that the output link rate is 4 Mbps.

Question 2

Consider the model of an output buffer in a packet switch for a 200 Mbps link. The arrival rate of packets to the queue is given by $\lambda = 400{,}000$ packets per second. Determine the average delay through the system and the average number of packets in the queue given:

(a) exponential packet sizes with an average length of 53 bytes
(b) fixed packet sizes of length 53 bytes
(c) packet lengths with a known mean of 53 bytes and a variance of 1600.

Question 3

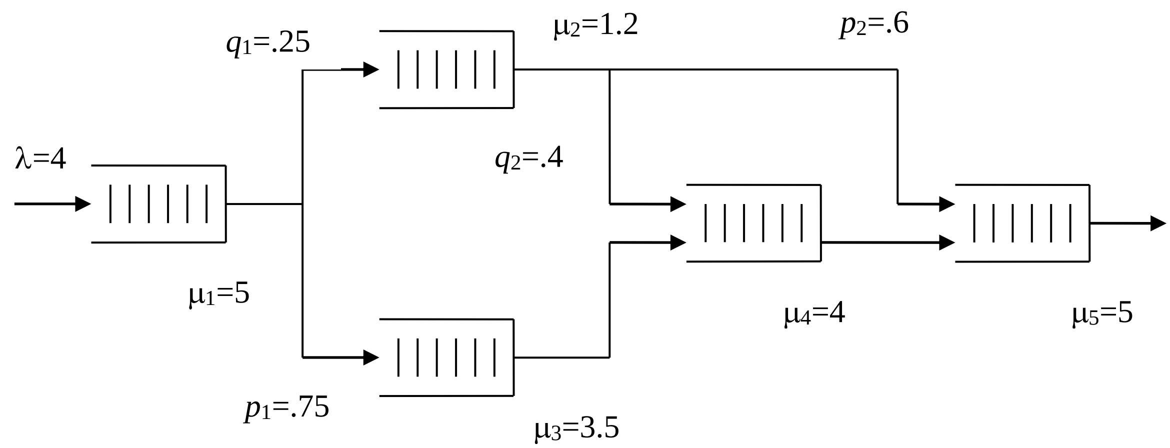
A queue is utilized in an ATM switch to control the packets using a link of rate 622 Mbps. Arrivals to the queue are Poisson and arrive at an average rate of $\lambda = 1.2$ packets/μsec.

(a) Given that ATM cells are fixed lengths of 53 bytes, determine the average time spent in the queuing system (i.e., including waiting time and service time).

(b) Assume that the designer of the system wants to ensure that the buffer allocated to the queue is capable of storing at least 5 times as many packets in the queue as the average number in the queue. What is the size of the buffer allocated in terms of the number of bits required to store the necessary packets?

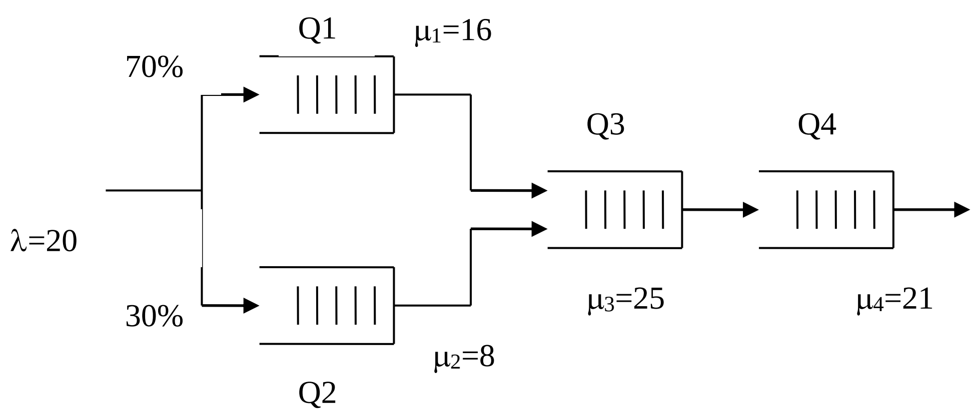
Question 4

Consider the following queueing system. All queues are M/M/1 and traffic streams split with the probabilities as given. Determine the expected delay through the entire system. Arrival rates are given as arrivals per second and service rates are given as departures per second.



Question 5

Consider the following queuing system. As indicated, 70% of the incoming packets enter Q1 and 30% enter Q2.



(a) Assuming that all queues are M/M/1, determine the average delay through the queuing system.

(b) Repeat part (a) assuming that Q1, Q2, and Q3 are M/M/1 and Q4 is an M/D/1 queue.

(c) Repeat part (a) assuming that Q1, Q2, and Q3 are M/M/1 and Q4 is an M/G/1 queue with the mean service time being 1/21 seconds and the variance of the service time being 0.005 seconds^2.

Question 6

Consider that a queue in a communications network is modeled as an $M/M/1/N$ queue where the size of the queuing system is $N = 3$.

(a) Assuming that the arrival rate is 10,000 packets per second and the average service time is 80 μsec:
   (i) Draw the state transition diagram that represents the behaviour of the queuing system.
   (ii) Determine the probability that a packet arriving at the queue must be discarded because the queue is full.
   (iii) Determine the average number of packets in the queue.
   (iv) Determine the average delay for packets that are not discarded.

(b) Assume now that $N = 2$. Given that the arrival rate is 10,000 packets per second, determine the maximum possible average service time so that the probability of a packet being discarded is less than 10^{-2}.

Question 7

A queuing system has Poisson arrivals with an arrival rate of 50 packets/μsec and two servers with an exponential service time of $t_s = .015$ μsec. In addition to the two servers, the system has one waiting position. Hence, the system is a M/M/2/3 queue.

(a) Draw the state transition diagram and determine the state probabilities.
(b) Determine the average number of customers in the queuing system.
(c) What is the probability of blocking?

Question 8

Consider a queuing system that behaves in the following manner:

- There are never more than two customers in the queuing system.
- The server must serve either two customers at a time or no customers.
- If a customer arrives and there are no customers in the queuing system, the customer will wait and will not be served immediately.
- If a customer arrives and there is one customer waiting and no customers being served, both customers will immediately begin to be served by the server.
- If a customer arrives and two customers are being served, the customer is discarded. (Hence, there can never be customers waiting while customers are being served.)
- The server serves two customers at once for the same length of time and the service time is exponentially distributed.

(a) Draw the state transition diagram for the system.
(b) Assuming that customer arrivals are Poisson at a rate of 20 customers per minute and the average time customers will take while being served (not including waiting time) is 2 seconds, what is the probability that a customer is blocked from being served (i.e., the probability that a customer is discarded)?

Question 9

A synchronous packet switch contains only input queues, with one first-come-first-serve queue at each input. During a switching cycle, one packet may placed in the queue and/or one packet may leave the queue. Packets at the head of the queue are kept in the queue until they can be successfully transmitted to the output link. That is, if the packet at the head of the queue is not blocked, it is removed from the queue. In any given switching cycle, the probability of a packet arriving at an input is 20% and the probability of blocking occurring for a packet at the head of the queue is 30%. The input queue can store up to 4 packets. Arriving packets are queue at the beginning of the switching cycle and departing packets are dequeued at the end of the switching cycle. Hence, an incoming packet must be queued before it is possible to dequeue the packet at the head of the queue. A packet must be placed in the queue for at least one switching cycle before it is transmitted. (For example, if a packet arrives in switching cycle *i* to an empty queue, it will be removed from the input queue no earlier than switching cycle *i*+1.)

(a) Draw the state transition diagram of the Markov chain formulation of the system.

(b) Determine the state probabilities.

(c) What is the probability that a packet will be discarded? Note that an arriving packet must be queued before the packet at the head of the queue can be dequeued.

ANSWERS:

Q1. (a) 0.426 (b) 0.928

Q2. (a) $\bar{T} = 13.95\,\mu\text{s}$ (b) $\bar{T} = 8.03\,\mu\text{s}$ (c) $\bar{T} = 11.40\,\mu\text{s}$

Q3. (a) $\bar{T} = 2.214\,\mu\text{s}$ (b) 5936 bits

Q4. (a) $\bar{T} = 6.17\text{ s}$

Q5. (a) $\bar{T} = 1.7\text{ s}$ (b) $\bar{T} = 1.224\text{ s}$ (c) $\bar{T} = 2.274\text{ s}$

Q6. (a) (ii) $P_B = 0.1734$ (iii) $\bar{k} = 1.225$ (iv) $\bar{T} = 122.5\,\mu\text{s}$
(b) $\bar{\tau} \leq 10.568\,\mu\text{s}$

Q7. (a) $P_0 = 0.468$, $P_1 = 0.351$, $P_2 = 0.132$, $P_3 = 0.049$
(b) average # packets = 0.762
(c) $P_B = 0.049$

Q8. (a) $P_0 = 3/8$, $P_1 = 3/8$, $P_2 = 1/4$
(b) $P_B = 1/4$

Q9. (b) $P_0 = 0.71436$, $P_1 = 0.25513$, $P_2 = 0.02734$, $P_3 = 2.928\times10^{-3}$, $P_4 = 2.51\times10^{-4}$
(c) $P_B = 2.51\times10^{-4}$