Performance Analysis of Cooperative Diversity Wireless Networks over Nakagami-\(m\) Fading Channel

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Abstract—This letter analyzes the performance of cooperative diversity wireless networks using amplify-and-forward relaying over independent, non-identical, Nakagami-\(m\) fading channels. The error rate and the outage probability are determined using the moment generating function (MGF) of the total signal-to-noise-ratio (SNR) at the destination. Since it is hard to find a closed form for the probability density function (PDF) of the total SNR, we use an approximate value instead. We first derive the PDF and the MGF of the approximate value of the total SNR. Then, the MGF is used to determine the error rate and the outage probability. We also use simulation to verify the analytical results. Results show that the derived error rate and outage probability are tight lower bounds particularly at medium and high SNR.

Index Terms—Average error rate, outage probability, cooperative diversity, amplify-and-forward, Nakagami-\(m\) fading.

I. INTRODUCTION

COOPERATIVE diversity networks technology is a promising solution for the high data-rate coverage required in future cellular and ad-hoc wireless communication systems. The cooperative diversity concept has been introduced in [1], [2], [3]. The basic idea of cooperative diversity networks is that (in addition to the direct signal from the source to the destination) some neighboring nodes can relay the signal of the source node to the destination node as shown in Fig. 1. As a result, the destination node can receive multiple independent copies of the same signal and can achieve diversity without the need to install multiple antennas at the source node or the destination node.

In [4] the authors derived asymptotic average symbol error probability for amplify-and-forward cooperative diversity networks. The resulting expressions derived in [4] (using the bounding approach) are general for any type of fading distributions provided the probability density function (PDF) of zero instantaneous SNR is not zero, which is not applicable for the Nakagami-\(m\) fading distribution.

In [5], [6] multi-hop relaying (not cooperative diversity) over Nakagami-\(m\) fading channels has been studied. The error rate is determined in [5] using a single integration of the conditional error probability \(P(\varepsilon|SNR)\) times the PDF of the SNR. Hence, it is not straightforward to extend the work in [5] to the case of cooperative diversity with \(M\) relay nodes (as shown in Fig. 1) because we have to use \(M + 1\) integrations in this case. Furthermore, their lower error bound is not tight enough at high SNR (see Figs.3 and 4 in [5]). In [6], the MGF of the SNR is used to find the bit error rate. Hence, it is straightforward to extend this work to the case of cooperative diversity with \(M\) relay nodes. However, the derived MGF in [6] is limited to the case of identical Nakagami-\(m\) fading channels.

In [7] the authors studied the performance of cooperative diversity networks over independent non-identical Nakagami-\(m\) fading channels and they found the MGF of the total SNR but their error probability bound is not tight enough at medium and high SNR (see Fig. 2 in [7]).

In this letter we derive a tight lower bound for the error rate and the outage probability of cooperative diversity networks over independent non-identical Nakagami-\(m\) fading channels with amplify-and-forward relaying and maximum ratio combining (MRC) at the destination node. We first find closed-form expressions for cumulative distribution function (CDF), PDF and MGF of the total SNR. Then, the MGF is used to determine the lower bounds. We verify the obtained analytical results using computer simulations. Results show that the derived error rate and outage probability bounds are tight bounds particularly at medium and high SNR.

II. SYSTEM MODEL

As shown in Fig. 1, a source node (S) and a destination node (D) communicate over a channel with a flat Nakagami-\(m\) fading coefficient (\(f\)). A number of cooperating nodes \(R_i\), \(i = 1, 2, \ldots, M\) relay the signal to provide the destination with multiple copies of the original signal. The channel coefficients between the source \(S\) and \(R_i\) (\(h_i\)) and between \(R_i\) and \(D\) (\(g_i\)) are also flat Nakagami-\(m\) fading coefficients. In addition, \(f\),

![Illustration of a cooperative diversity wireless network.](image-url)
\[ h_i \text{ and } g_i \text{ are mutually-independent and non-identical. We also assume here without any loss of generality that all additive white Gaussian noise (AWGN) terms have zero mean and equal variance } N_0. \text{ Assuming MRC at the destination node, the total SNR at the destination node can be written as } [4], [5], [6], [7] \]

\[ \gamma_{equ} = \gamma_f + \sum_{i=1}^{M} \gamma_{hi} + \gamma_{gi} + 1 \]  

(1)

where \( \gamma_{hi} = h_i^2 E_s / N_0 \) is the instantaneous SNR between \( S \) and \( R_i \), \( \gamma_{gi} = g_i^2 E_s / N_0 \) is the instantaneous SNR between \( R_i \) and \( D \), \( \gamma_f = f^2 E_s / N_0 \) is the instantaneous SNR between \( S \) and \( D \) and \( E_s \) is the signal transmitted energy.

### III. PERFORMANCE ANALYSIS

#### A. Error Performance

We derive in what follows a tight lower bound on the average error performance. The total SNR can be approximated by its upper bound \( \gamma_b \) as follows

\[ \gamma_{equ} \leq \gamma_b = \gamma_f + \sum_{i=1}^{M} \gamma_i \]  

(2)

where \( \gamma_i = \min(\gamma_{hi}, \gamma_{gi}) \). The approximate SNR value \( \gamma_b \) is analytically more tractable than the exact value in (1); as a result, this facilitates the derivation of the SNR statistics (CDF, PDF, MGF). This approximation is adopted in many recent papers (e.g., [8], [9]) and it is shown to be accurate enough at medium and high SNR values as will be discussed in the results section.

Assuming the independence of \( \gamma_{hi}, \gamma_{gi} \), and \( \gamma_f \) the MGF of \( \gamma_b \) can be written as

\[ M_{\gamma_b}(s) = \prod_{i=1}^{M} M_{\gamma_i}(s) \]  

(3)

where \( M_{\gamma_f}(s) \) and \( M_{\gamma_i}(s) \) are the MGF of \( \gamma_f \) and \( \gamma_i \), respectively. Using the definition of the MGF as \( M_{X}(s) = \mathbb{E}(e^{-sX}) \) (where \( \mathbb{E} \) is the statistical average operator), it can be easily shown that

\[ M_{\gamma_f}(s) = \left(1 + \frac{\gamma_f}{m_f} s\right)^{-m_f} \]  

(4)

where \( m_f \) is the Nakagami-m fading parameter of \( f \) and \( \gamma_f = f^2 E_s / N_0 \). In order to find \( M_{\gamma_i}(s) \), we find the CDF of \( \gamma_i \) as follows

\[ F_{\gamma_i}(\gamma) = 1 - P(\gamma_{hi} > \gamma) P(\gamma_{gi} > \gamma) \]

\[ = 1 - \frac{\Gamma(m_{hi} / \gamma_{hi}) \Gamma(m_{gi} / \gamma_{gi})}{\Gamma(m_{hi} + m_{gi})} \]  

(5)

where \( \Gamma(\cdot) \) is the incomplete gamma function [10, eq.(8.350.2)], \( \Gamma(\cdot) \) is the gamma function [10, eq. (8.310.1)], \( \gamma_{hi} = \mathbb{E}[(h_i^2 E_s / N_0)] \), \( \gamma_{gi} = \mathbb{E}[g_i^2 E_s / N_0] \), and \( m_{hi} \) and \( m_{gi} \) are

\[ M_{\gamma_i}(s) = \left(\frac{m_{hi}}{\gamma_{hi}}\right)^{m_{hi}} \left(\frac{m_{gi}}{\gamma_{gi}}\right)^{m_{gi}} \frac{\Gamma(m_{hi} + m_{gi})}{\Gamma(m_{hi}) \Gamma(m_{gi})} \left[1 - 2 F_1 \left(1, m_{hi}, m_{gi}; m_{hi} + m_{gi}; 1; \frac{m_{hi}}{\gamma_{hi}}, \frac{m_{gi}}{\gamma_{gi}}\right) \right] 

+ \frac{1}{m_{hi}} 2 F_1 \left(1, m_{hi}, m_{gi}; m_{hi} + m_{gi} + 1; \frac{m_{hi}}{\gamma_{hi} + m_{gi} / \gamma_{gi}}, \gamma_{hi} + \gamma_{gi}\right) \times \frac{1}{\gamma_{hi} + \gamma_{gi}} \]  

(7)

Finally, \( M_{\gamma_b}(s) \) can be calculated with the help of [10, eq.(6.455.1)] as Eqn. 7 on the bottom of this page, where \( 2 F_1(\cdot, \cdot) \) is Gauss’ hypergeometric function defined in [10, eq. 9, 100]. If \( m_{hi} = m_{gi} = m_{hi} \) and \( \gamma_{hi} = \gamma_{gi} = \mu \) it can be shown that (7) greatly simplifies to the form

\[ M_{\gamma_b}(s) = \left(\frac{m_{hi}}{\mu}\right)^{2 m_{hi}} \frac{\Gamma(2 m_{hi})}{m_{hi}^2 \Gamma(m_{hi})} \frac{2}{\left(\frac{m_{hi}}{\mu} + s\right)^{2 m_{hi}} + 2 F_1 \left(1, 2 m_{hi}; m_{hi} + 1; \frac{m_{hi}}{\mu} + s\right)} \]  

(8)

By substituting (4) and (7) into (3) we obtain a closed form expression of \( M_{\gamma_b}(s) \). Using \( M_{\gamma_b}(s) \), the error rate can be evaluated for a wide variety of M-ary modulations (such as M-ary phase-shift keying (M-PSK) and M-ary quadrature amplitude modulation (M-QAM)) [11]. For instance, the average symbol error rate (SER) for M-PSK can be written as [11, p. 271]

\[ P(e) = \frac{1}{\pi} \int_0^{\infty} M_{\gamma_b}(\frac{g_{PSK}}{\sin^2(\theta)}) \, d\theta \]  

(9)

where \( g_{PSK} = \sin^2(\frac{\theta}{2}) \). The SER in (9) can be evaluated using a single integral and can be done with simple numerical integration techniques. Furthermore, (9) can be upper bounded by a simple form: \( P(e) \leq (1 - 1/M) M_{\gamma_b}(g_{PSK}) \) [11, p.275].

#### B. Outage Performance

The outage probability \( P_{out} \) is defined as the probability that the instantaneous total SNR falls below a given threshold value \( \gamma_0 \). \( P_{out} \) can be calculated using the MGF [11, Ch. 1] as follows

\[ P_{out} = \Im^{-1}(M_{\gamma_b}(s)/s) \bigg|_{\gamma_0} \]  

(10)

where \( \Im^{-1}(\cdot) \) denotes the inverse Laplace transform. The inverse Laplace transform can be done analytically or using simple numerical techniques as in [12], [13].

### IV. Numerical Results

In this Section, we show numerical results of the analytical bit error rate (BER) and \( P_{out} \) for binary phase shift keying (BPSK) modulation. We plot the performance curves in terms of average BER and outage probability versus the SNR of the transmitted signal \( (E_s/N_0) \) dB where \( E_s \) is the transmit power.
Fig. 2. BER performance for arbitrary $E(f^2)=1$, $E(h_i^2)=0.75$, $E(g_i^2)=0.5$, several numbers of cooperative paths ($M = 0, 1, 2, 3$) and $m_f = m_h_i = m_g_i = 0.5$.

Fig. 3. Outage performance for arbitrary $E(f^2)=1$, $E(h_i^2)=0.75$, $E(g_i^2)=0.5$, several numbers of cooperative paths ($M = 0, 1, 2, 3$) and $m_f = m_h_i = m_g_i = 0.5$.

energy signal. We also show computer simulation results for verification.

Fig. 2 shows the BER performance at different values of the number of cooperating nodes ($M$). We also plotted in Fig. 2 the BER bound derived in [4] for comparison. It is clear that the BER bound derived in this letter is much tighter than that proposed in [4], especially for medium and high $E_s/N_0$ values. For instance, for $M = 3$ and $E_s/N_0 = 15$ dB, the exact BER (from simulation) is equal to $8 \times 10^{-4}$, while the BER lower bound from [4] is equal to $2.1 \times 10^{-4}$ and our lower bound (from (9)) is equal to $6 \times 10^{-4}$. This trend (the tightness of our bound compared with that of [4] at medium and high $E_s/N_0$ values) is valid at different values of $M$ as shown in Fig. 2. From Fig. 2, we can also notice, as expected, that the number of cooperating relays ($M$) has a strong impact on performance enhancement and the achieved diversity order.

Fig. 3 illustrates the outage performance determined from (10). It is clear again that the difference between the exact (from simulation) and analytical results (lower bound) for $P_{\text{out}}$ is small for medium and high values of $E_s/N_0$. It should be noted that for Figs. 2 and 3 the tightness of the error performance improves as $E_s/N_0$ increases; however, the proposed lower bound (for BER and $P_{\text{out}}$) slightly loses its tightness at low $E_s/N_0$ values particularly when $M$ increases. This is due to the fact that the accuracy of total SNR approximation (in (2)) improves as $E_s/N_0$ increases.

V. CONCLUSIONS

Performance bounds for cooperative diversity networks over independent non-identical Nakagami-$m$ fading channels have been investigated. The total SNR is approximated by its upper bounded. Then, closed-form expressions for the MGF, PDF, and CDF of the approximate total SNR have been derived. The MGF is used to determine lower bounds for the error rate and outage probability. Our numerical results show that derived error rate and outage probability are a tight lower bound particularly at medium and high SNR.

REFERENCES