Equation (3–59) was obtained by eliminating $\dot{x}$ from Equations (3–57) and (3–58). Equation (3–60) was obtained by eliminating $\theta$ from Equations (3–57) and (3–58). From Equation (3–59) we obtain the plant transfer function to be

$$\frac{\Theta(s)}{-U(s)} = \frac{1}{Mls^2 - (M + m)g}$$

$$= \frac{1}{Ml \left( s + \sqrt{\frac{M + m}{Ml}} \frac{g}{\sqrt{g}} \right) \left( s - \sqrt{\frac{M + m}{Ml}} \frac{g}{\sqrt{g}} \right)}$$

The inverted pendulum plant has one pole on the negative real axis [$s = -(\sqrt{M + m/\sqrt{Ml}})\sqrt{g}$] and another on the positive real axis [$s = (\sqrt{M + m/\sqrt{Ml}})\sqrt{g}$]. Hence, the plant is open-loop unstable.

Define state variables $x_1, x_2, x_3,$ and $x_4$ by

$$x_1 = \theta$$
$$x_2 = \dot{\theta}$$
$$x_3 = x$$
$$x_4 = \dot{x}$$

Note that angle $\theta$ indicates the rotation of the pendulum rod about point $P$, and $x$ is the location of the cart. If we consider $\theta$ and $x$ as the outputs of the system, then

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \theta \\ x \end{bmatrix} = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}$$

(Notice that both $\theta$ and $x$ are easily measurable quantities). Then, from the definition of the state variables and Equations (3–59) and (3–60), we obtain

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{M + m}{Ml} x_1 - \frac{1}{Ml} u$$
$$\dot{x}_3 = x_4$$
$$\dot{x}_4 = \frac{m}{M} g x_1 + \frac{1}{M} u$$