1 Introduction

This project is intended to give you an opportunity to apply the theory that has been given in lectures. Follow the instructions in the succeeding sections. The deliverable should be in a technical report form, is intended to be original work and should be carried out individually. If you use any resources other than your notes or textbook, provide references.

You will be required to use a combination of hand calculations, MATLAB calculations, simulations and plots. Show all of your workings, and MATLAB scripts must be attached in an Appendix. Ensure that your code is properly commented.

The answers to the questions in the following sections will form the content of your report: each question should be a section.

Question 1.1. Hard Disk Drive Modeling

A hard disk drive (HDD) must control the position of the read-write (R/W) head very precisely in the presence of possibly extreme shock loads and wideband disturbances \[1\]. The basic model for the positioning arm of the HDD is illustrated in Fig. 1.

The arm on which the read-write head is mounted is controlled by a voice coil motor (VCM) actuator that pivots the arm about point \(P\). The VCM has a torque constant \(K_t = 3.25 \times 10^3 \, N \cdot \mu m/Volt\) and the moment of inertia of the R/W head is \(J_a = 5.0771 \times 10^{-5} \, kg \cdot \mu m^2\). The position and velocity of the head over the disk are \(x \, \mu m\) and \(\dot{x} \, \mu m/s\) respectively. The VCM applies a control torque \(\tau_c \, N\) to the arm to displace it by angle \(\theta\)\[^1\]. The VCM input is the voltage \(v(t) \, Volts\).

1. Write a differential equation that models the system dynamics by summing the torque \(M_z\) around point \(P\). Then derive a second-order state-space model for the system where the state vector is \(x = [x \, \dot{x}]^T\), the input to the system is \(v\) and the output is position of the R/W head on the disk \(x\, \mu m\).

\[^1\]For small angles \(\theta\) it can be assumed that \(\sin \theta \approx x\)
2. Find an expression for the open loop transfer function of this system.

3. Is the system controllable?

**Question 1.2. State Feedback Controller Design**

We wish to design a regulator for the system of Q1.1 that will hold the head stationary over the disk.

1. Design a state feedback gain for the regulator such that the closed-loop poles will be at \( \mu_1 = -8000, \mu_2 = -10000 \)

2. Design a state feedback gain for the regulator such that the closed-loop poles will be at

\[
\begin{align*}
\mu_1, \mu_2 &= 1 \times 10^5 \times \left( -\sin \frac{\pi}{3} + \cos \frac{\pi}{3} j, -\sin \frac{\pi}{3} - \cos \frac{\pi}{3} j \right) \\
&= \left( -\sin \frac{\pi}{3}, -\cos \frac{\pi}{3} \right) j
\end{align*}
\]  

(1)

3. Test the closed-loop response of each design to a unit impulse and a unit step, plotting the results. In your opinion, Which regulator is better, and why?

4. Design your own state feedback regulator and compare the results to the previous controllers. Can you do better?

5. Tabulate the results of all three controllers. The table should contain quantities for rise time, settling time, overshoot and undershoot.\(^2\)

\(^2\)See MATLAB command stepinfo()
**Question 1.3. Augmented Model**

The simple model of Q1.1 unfortunately does not include the resonant mode of the arm. The structural flexibility of the arm can cause oscillation in the R/W head. The model for the arm’s resonant model will be referred to as the “high frequency” (or HF) model. In order to design a realistic controller for this system, we will need to include the effects of the arm resonance. The transfer function is as follows:

\[
G_{hf}(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2}
\]

where \(\omega_n = 1.5707 \times 10^4 \text{ rad/s}\) and the damping is \(\zeta = 8 \times 10^{-2}\).

1. Find the state space model for the HF system in controllable canonical form, where the input and output are the same as in Q1.1, and the state vector is \(x = \begin{bmatrix} x_{hf} \\ \dot{x}_{hf} \end{bmatrix}^T\).

2. Identify the location of the open loop poles of this resonant model, and plot the unit step and impulse response of this system.

3. Augment the low frequency state space model of Q1.1 by combining it with the high-frequency resonant model. The resulting (total) system model will be fourth order \((n = 4)\), and should have state vector

\[
x = \begin{bmatrix} x_{lf} \\ \dot{x}_{lf} \\ x_{hf} \\ \dot{x}_{hf} \end{bmatrix}
\]

where \(x_{lf}(t)\) is equivalent to \(x\) in the simple model. The new output of the system is simply the sum of the two separate models \(y(t) = x_{lf}(t) + x_{hf}(t)\).

4. Find an expression for the transfer function of the augmented system

\[
G_{tot}(s) = \frac{Y(s)}{U(s)}
\]

**Question 1.4. State Feedback Controller Design for Augmented Model**

As before, we wish to design a regulator for the augmented system model of Q1.3 that will hold the R/W head stationary over the disk.

1. Design a regulator in which the poles of the HF system are not moved, but the remaining two poles are moved to \(\mu_1 = -8000, \mu_2 = -10000\). Plot the response of the closed loop system to unit step and impulse inputs.
2. Design a regulator in which the poles of the HF system are not moved, but the remaining two poles are moved to

\[ \mu_1, \mu_2 = 1 \times 10^5 \times \left( -\sin\frac{\pi}{4} + \cos\frac{\pi}{4} j, -\sin\frac{\pi}{4} - \cos\frac{\pi}{4} j \right) \]  \hspace{1cm} (3)

Plot the response of the closed loop system to unit step and impulse inputs. Describe how the arm’s resonance affects the closed-loop response of the system.

3. Design a regulator that places the HF system poles \(^{\footnote{3}}\) and places the other two poles at the locations specified by Eqn\(^{\footnote{3}}\). Plot the response of the closed-loop system to unit step and impulse inputs.

4. Try your own regulator design (decide on your own closed-loop pole locations).

5. Tabulate the results for the regulators of \(^{\footnote{2-4}}\). The table should contain quantities for rise time, settling time, overshoot and undershoot \(^{\footnote{4}}\).

**Question 1.5. Type 1 Servo Controller**

In the previous question, we considered a regulator design for the HDD R/W head control. We were primarily interested in how well the control system rejected disturbance (which was referred to the input \(u\)) and thus how well it regulated the state \(x(t) = 0\).

1. Calculate the state feedback gain for a Type 1 servo design for the augmented system model of \(^{\footnote{Q1.4}}\). Use your own design (closed-loop pole locations).

2. Plot the response for the type 1 servo to a unit step input.

3. Plot the VCM actuator voltage \(u(t)\) for the unit step input.

4. Sketch a state-space model diagram for the Servo controller. Comment on the implementation of the system. How might the state be measured? Are the state feedback gains practical?

**References**


\footnote{3}{HINT: Do not change the frequency of the poles, but increase the damping.}

\footnote{4}{See MATLAB command stepinfo()}