

ENGINEERING 9051

Ice as a Load Bearing Medium

February 1994

For Dr. I. Jordaan
Dr. J. Molgaard

By Stephen Bruneau

Abstract

The reaction of a given floating ice sheet to an applied load varies significantly with the rate of load application. Further, load concentration, ambient temperature, ice purity, porosity, continuity and even loading history influence the response of the ice layer. Theoretical models have been tailored to model the behaviour of ice, yet, its precise nature in given situations remains uncertain. This is, in part, unavoidably due to the relevance of naturally varying site specific parameters and the complex natural processes that influence ice generally. The practical result is that most people requiring bearing capacity information rely on simple, semi-empirical guidelines and personnel experience. Unfortunately, associated with these subjective measures is some risk and, often, design inefficiencies.

The following paper assess the safety of a lake as a load bearing medium. Part I reviews the background theory and practice relevant to the treatment of ice as a load bearing medium. In Part II these design principles are used to develop operational guidelines for a hypothetical problem. The author has given attention to a case study which requires one to consider bearing resistance to static (long-term), quasi-static (short term) and dynamic loading under a few temperature regimes.

PART I

Background: Theory and Practice

General

Ice can roughly be categorized as a visco-elastic crystalline material. Since it occurs naturally very near its melting point, it behaves, paradoxically, as a brittle solid in some conditions and as a viscous fluid in others. In general, though, when ice is subjected to a stress it initially deforms in three distinct ways: it undergoes an immediate elastic strain, a transient time-dependent delayed elastic strain and time-dependent non-linear viscous creep strain (Sanderson, 1988).

Sinha (1983), whose work is broadly accepted, has formulated constitutive laws for columnar S2 ice (most common and relevant here) based on the above characterization. For uniaxial load testing they may be written as follows:

- 1/ The immediate elastic strain is found by the ratio of stress to Young's modulus (Hooke's Law):

$$\epsilon_e = \sigma/E$$

- 2/ The delayed elastic strain is written as

$$\epsilon_d = C_1 * (d_1/d) * (\sigma/E)^s * (1 - \exp[-(a_T * t)^b])$$

- 3/ The viscous term or secondary creep strain is may be written as

$$\epsilon_v = \dot{\epsilon}_{v_1} * t * (\sigma/\sigma_1)^n$$

where E is Young's modulus (≈ 9.5 GPa), σ is uniaxial stress, C_1 is a constant ($= 9 \times 10^{-3}$ m), d_1/d is a representative grain diameter, s is a constant (≈ 1), a_T is a quantity dependant upon temperature T ($= 2.5 \times 10^{-4} \text{ s}^{-1}$), t is time, b is a constant ($\approx 1/3$), $\dot{\epsilon}_{v_1}$ is secondary creep strain rate and σ_1 is a constant stress value and n is a constant (≈ 3). The total strain for a given stress as a function of time for the above formulas is plotted in Figure 1.

Bending

The basis for theoretical bearing capacity models can be summarised by reviewing the case for a simple beam in flexure. The deflection of a loaded beam can be characterized by an instantaneous and recoverable elastic strain followed by varying degrees of creep depending upon strain rate and load duration similar to that illustrated above for the uniaxial constant stress test. In field

trials it has been observed that deformed ice beams substantially recover when unloaded (Michel, 1978). Fracture is catastrophic for the beam due to the first crack in the lower extreme layer - a similar finding as uniaxial testing in tension. Michel believes that no tertiary creep mechanisms are present in pure bending and as a result no accelerated creep will occur near failure.

Properties Affecting Bending

The flexural strength of an ice beam may be expected to be obtained from uniaxial tensile strength at the bottom layer of the beam, which further depends upon the strain rate in the ductile range and on the degree of plastification of the ice section. Under flexural stress freshwater ice will behave as follows:

$\dot{\epsilon} > 10^{-3} \text{ s}^{-1}$	Elastic with brittle fracture
$10^{-3} > \dot{\epsilon} > 10^{-5} \text{ s}^{-1}$	Partial plastification of the section with brittle fracture
$10^{-5} > \dot{\epsilon} \text{ s}^{-1}$	full plastification of section with permanent creep.

For brittle fracture conditions tensile strength of St. Lawrence river ice S2 is approximately 1MPa on the top layers and .5 MPa on the bottom layers, in compression for similar ice the values are 1.3 MPa and .54 MPa respectively. For S2 freshwater ice $E = 9.27 \cdot (1 - 1.36 \times 10^{-3} \text{ } ^\circ\text{C}) \cdot 10^9 \text{ Pa}$ which illustrates its small variation with temperature ($^\circ\text{C}$). Porosity and density have a much greater influence on ice strength. This is apparent when brine pockets which occupy portions of the ice lattice in sea ice significantly reduce its strength.

Ice Covers

In making the transition from beams to planar ice covers the material properties of ice do not change. The failure mechanisms do, however, and this is largely due to the three dimensional support boundary conditions. These include lateral and longitudinal stress distributions, and, an underlying hydrostatic pressure on the ice sheet. If, according to Ashton (1986), an additional load is placed on an ice sheet it will sink until the hydrostatic pressure balances with the load. Thus loads are largely supported by the water and the ice sheet merely governs the area over which a load is distributed. Of course to distribute the load the ice sheet must deform and in doing so stresses are generated.

Under stress a given ice layer will first deflect elastically. Increased loads cause radial cracks to form at the bottom of the

ice layer. These cracks then extend radially and vertically to the top, new radial cracks are also formed. When the load is further increased to double that which caused the initial cracking, circumferential cracks form at a characteristic length from the application point. This is followed by the collapse of the "dish" shaped ice layer. If the ice is rotten as is often the case in Spring, the failure may arise prematurely due to shear immediately adjacent to the load. See Figure 2.

Sinha (1992) states that a rapid but time dependent recovery occurs on lake ice after the removal of an applied load. Ice exhibits the elastic, delayed elastic and permanent deformation depending on load conditions, in a similar way to lab specimens and beams.

The theoretical solution to the equations for the elastic deflection of a plate on an elastic foundation was obtained by Hertz (1884). It was determined that at the origin (under the load) the deflection is found by

$$w_{\max} = \frac{Pl^2}{8D}$$

$$l = 4\sqrt{\frac{D}{k}}$$

$$D = \frac{Eh^3}{12(1-\nu^2)}$$

where D denoted the flexural rigidity, l is the characteristic length (action radius) of the ice layer, k is 9810 for freshwater, ν is Poisson's ratio (1/3), and P is the load in N.

Frederking and Gold (1976) proposed several formula which characterize the deflection rate as a function of stress levels for long term storage on ice covers. They concluded that the time for deflection to reach the freeboard limit can be computed with the total instantaneous and delayed elastic deformation which increases logarithmical with a decrease in load. Ashton (1986) admits however, that predicting response to long term load situations such as parked vehicles are very difficult to analyze. There is no reliable and simple method for doing so without field measurements.

Common Practice

Several philosophies have been applied to the question of safe bearing capacity of level ice. Michel (1978) strongly defends the use of "first crack" initiation to specify the ice capacity whereas

Gold (1900) prefers the allowance of cracks but with low risk of breakthrough. Breakthrough requires roughly twice the load of first crack formation. Kerr (1976) developed procedures for determining failure loads (see Table 2 and note that P_f is failure load and t_f is "safe" storage time of a load). Most will agree that criteria should be based on a combination of the importance of the load bearing activity (emergency use vs recreational use), the duration of the load condition (storage vs transport) and availability of information (survey of cracks number of thickness measurements, etc).

In general, those requiring ice bearing capacity information are not ice physicists or engineers. They are town officials, practitioners, transport authorities and the general public. Effective appraisal of ice thus demands lucid guidelines and simple formulae. In view of this and the many factors which influence the bearing capacity of ice which cannot be systematically included in computations, the overall bearing capacity has often been simplified to the following formula:

$$P=A*h^2$$

Where h is the consolidated ice thickness, A is a constant determined from ice cover observations and P is the load. The most useful and practical information on bearing capacity of ice covers was accumulated by Gold of NRCC (Gold, 1971) on the performance of fresh water ice covers. The information reviewed in Figure 3 pertains to recorded failures of isolated vehicles, where both weight of vehicle and ice thickness were recorded. The load under which no breakthroughs (catastrophic failure) occurred is:

$$P=2.0*10^4*h^2$$

where P is in Kg and h is in meters.

Michel (1978) recommends that the working stress of an ice cover be limited to that which causes the formation of first lower-ice-level radial cracks. He also incorporates the loading condition into his formula as follows:

$$P-\beta*\sigma_f*h^2$$

where $\beta = .5$ for dynamic loading and σ_f is the flexural strength of ice in Pa and P is the load in N. Note that values of β may be as high as 1.4 for breakthrough criteria of highly plasticized ice. He concludes that this formula can be effectively used provided σ_f and h are determined in the field and that β is computed from the loading condition. Without a proper field survey, Michel states, bearing capacity should be limited to that defined by no risk of breakthrough above.

Note that a combined value of $\beta \cdot \sigma_f$ of 3.5×10^4 has been used in Canada for freshwater ice and is recommended by Transport Canada (1976) for aircraft landing criteria. Russian Red Army guidelines (1946) suggest that the combined value of 7×10^4 for wheeled vehicles and 1.23×10^5 for tracked vehicles can be used in everyday conditions.

Nevel and Assur (1968) analyzed the bearing capacity of lake ice when crowds gather into varying patterns. Both uniform strip and circular loading conditions were appraised. For all practical purposes the ice thickness required for both strip and circular loading patterns simplified to

$$h = 3/8 * \left(\frac{E}{\rho} \right) * \left(\frac{q}{\sigma} \right)^2$$

where q is the loading intensity, E is the modulus of elasticity ρ is the density of water and σ is the maximum tensile stress (flexural strength). Poisson's ratio, μ , is assumed to be $1/3$. They noted that under marginal conditions dense crowds should be avoided and that periods of prolonged thaw may reduce capacity considerably for a given ice sheet.

Sinha (1992) concluded that up to 1 tonne/m^2 distributed over an area with a diameter 10 times greater than the ice thickness could be supported by a $.65 \text{ m}$ thick cold ice cover (to a deflection of approximately 10 percent of the ice thickness which is the freeboard). If the criteria of ice flooding were to be overlooked as would be the case for shorter loading durations then a larger load could be stored. This is a creep related phenomena in which the deflection is controlled by a creep strain rate and load duration. In the case of Sinha's work he applied a 36 ton load over a 36 m^2 area yielding the following:

$$A = \frac{P}{h^3} = 36000 / (.65)^2 = 8.5 \times 10^4 > 3.5 \times 10^4$$

This is well beyond the first criteria for loading based on radial crack formation (Michel) $A = 2.0 \times 10^4$ and Transport Canada's $A = 3.5 \times 10^4$ for low risk of breakthrough (shown). But since there is no record of ice cracks in Sinha's paper we can only assume that they were not recorded or that none occurred due to the broad distribution of the load, cold temperatures, and the continuity of the ice cover.

Michel categorises vehicular speeds (v) on ice by the state of plastification that they results. They are listed as follows:

$v > 3\text{m/s}$	full elastic behaviour
$3\text{ m/s} > v . 0.03\text{ m/s}$	partial plastification of the ice
$0.03\text{ m/s} > v$	full plastification of the ice

He further notes the significance of vehicular speed on the production of waves in the ice. The critical speed of wave propagation in an ice cover has been determined computationally by Nevel(1970) and is well documented in Michel(1978). To avoid resonance and wave amplification vehicles should avoid travelling at this speed.. Since the critical speed varies with depth and lies well within the range of typical vehicular speeds special precautions should be undertaken that would avoid problems. These include varying speed continuously maintaining a relatively low speed at all times and avoid travelling parallel or perpendicular to the shore.

PART II CASE STUDY

Introduction

The community of Pigeon Inlet, Nf. would like to start a curling club. Unfortunately, they do not have a curling facility and do not have the funds to build one, at least indoors. During a recent council meeting the suggestion arose that they consider an interim measure while funds are being raised for a permanent facility. The suggestion was that a curling rink be established during the winter months on Pigeon Pond, 2 km North of the community. It was decided that an engineering study would be undertaken to investigate the said utilization of Pigeon Pond and to establish a preliminary design for an operational facility.

Design Information

Pigeon Pond is 1 km long, 0.5 km wide, oval in shape and has tapered sandy banks which slope towards the deepest part of the pond at 10m (see Figure 4). A ramp for launching recreational boats connects the pond to Pigeon Road which runs roughly parallel to the pond's long axis. There is a single stream inlet on the north end of Pigeon Pond and a single outlet to the South which is the back-up water supply for Pigeon Cove. The cold temperatures in the winter result in complete ice cover typically from November to May. There are 800 people in Pigeon Cove, most drive cars and pick-up trucks.

A curling rink requires an ice temperature around -5 at the surface. Rinks are approximately 150 feet (45.7m) long and 14.5 feet (4.4m) wide. They are flat (no camber) with a surface roughness (pebble) applied for better frictional characteristics (to be discussed later). Rinks are constructed in two phases, the first involves the laying of a thin (1/4 inch, 6mm) smooth ice layer over which is painted the game lines. The second ice layer is then applied (a thickness of about 1 inch, 25 mm). The pebble is applied with a special hand-held sprinkler mechanism which requires great skill to properly operate. The air temperature in curling rinks varies with the level of human activity but is usually kept between -3 to 4 degrees.

Proposed Design

A facility for the center of the pond has been planned. It requires parking for 120 vehicles (cars and pick-ups) and some 50 spaces for skidoos. The maximum weight of any one of these vehicles is 3 tons (2 ton truck with 1 ton payload). Modest bleachers and chairs to accommodate a maximum of 300 people are to be installed. The rinks and the seating area are to be covered with a temporary bubble-type structure for shelter and temperature control. The bubble-type

structure (often used for covering tennis courts in winter) is 50mx30m and requires one-half ton ballast per meter perimeter for structural pressure and stability. An ice road is required to link an on-ice parking lot with the main road. Parking spaces are sized in accordance typical maneuvering guidelines plus consideration for load distribution. Snow clearing is required for the parking and road infrastructure. Figure 5 shows the layout of the facility.

The engineering assessment has found that the boat ramp is capable of supporting the vehicular loads provided only one at a time pass from the lake ice to the main road. The support ballast for the bubble structure is to be provided by $1/2 \text{ m}^3$ barrels full of ice (water pumped from the lake). All barrels are to be gathered in a hexagonal pattern approximately every 7 meters around bubble to accommodate the tie-down requirements.

Two rinks are to be constructed using 4x4 timbers as retaining structures for the 1.5 inch flooded playing surface. Small bleachers and seats are to be arranged in the pattern around the rinks as indicated in Figure 4.

A list of loads, conditions and recommendations are given in Table 1. The cases that dominate design considerations are the long term storage of ballast on the ice and the movement of trucks on the roadway (the proximity of large parked vehicles can be controlled). Of these the most significant loading condition appears to be that of the structural ballast. It is the heaviest and most permanent load condition. Since the facility is strictly recreational and is dominated by human activity, the criteria chosen for safe ice thickness in most cases is the most conservative measure, ie. the zero breakthrough condition discussed above.

An appraisal of the performance of this criteria against creep for long term storage of the ballast is reviewed in Figure 6. Under the value of unity (unloaded freeboard/loaded deflection = 1.0 is also plotted) one runs the risk of flooding the ice surface. It can be shown that Sinha (1992) experienced approximately 80% more deflection than theoretical values computed using Hertz's formula. At the recommended ice thickness of at least 0.42 m it can be seen that the freeboard (8%-10% of h) is not attained for the Pigeon Pond case study and that a safety factor of 5 exists. This factor is sufficient to cover the degree of error observed for Sinha plus some additional deflection which will result from a longer load duration. As a precautionary measure it is recommended that the zero breakthrough criteria be used and that a monitoring program is included to watch for very long term deflections.

Guidelines for the safe operation of the facility are summarized below.

Operational Guidelines:

- The safe ice thickness is to be calculated using the following formula:

$$h \geq \sqrt{\frac{P}{2.0 \times 10^4}}$$

where h is the ice thickness in meters and P is the load in Kg. For the case of the ballast the ice thickness required is 0.42 m. Ice thicknesses are to be taken every three to four days. After extreme events (sudden rise in temperature - extreme wind or long sunny spells) new tests should be conducted.

- The flexural strength is to be determined in the field bi-weekly using a standard beam test for checking against the criteria above. It is to be used in the formula:

$$h \geq \sqrt{\frac{P}{\beta * \sigma_f}}$$

where σ_f is in Pa, β is .05/9.81 and p is in Kg. Use the lesser of the two h's computed above.

(Note: Since the fresh water in Pigeon Pond is potable it may be considered pure, thus, only tensile strength of ice need be tested. Tensile strength depends little on temperature and mostly on thickness and crystallographics determined in-situ so the ice should be surveyed for both periodically.)

- Avoid close encounters with the inlets and outlets where dangerously thin ice may exist.
- Holes should be spaced at 30 m for ice thickness determination and only continuous dense ice should be counted in the effective ice thickness measurements.
- The pond ice should be tested for thickness via a coring device (cylindrical auger) and air temperature is to be taken daily and recorded.
- Where wet cracks are observed (water observed upon formation and not yet fully refrozen) semi-infinite plate analysis states that capacity is to be reduced to 1/2 the value for a continuous sheet. Where wet cracks run both directions then reduce the capacity by 75%. A foot survey of the pond every morning to track the formation of new wet cracks is required.

- Leave 5-10cm of snow on all level ice (minus the rinks) to protect it from deterioration due to solar radiation and to avoid slippage of vehicles and pedestrian traffic.
- Snow banks may cause longitudinal cracking due to loading so spread them out evenly and avoid high piles. If too much accumulates > 1.5m then remove the snow to a distant location (200m+).
- Bare ice thickens and becomes more buoyant than snow-covered ice. This can cause longitudinal cracks also so avoid differential insulation of the ice surface.
- Freshwater ice strength is almost temperature independent but deteriorates rapidly in direct solar radiation. It dislocates crystals at their boundaries and forms candle ice which has only a fraction of the bearing capacity of solid ice. Curtail all activity on the pond if Warm (> -2 degrees) sunny weather prevails for two days or more. If temperature goes above +4 degrees suspend activity immediately.
- Since the depth of Pigeon Pond goes from 0-10m ice wave resonance can be a problem where vehicle speeds approach the critical wave propagation speeds. Keep speeds below 20 kph and continuously vary speed from 10 to 20 kph. Also, do not approach the shore ramp normal to the shoreline and avoid travelling parallel to the shore. Both may give rise to dynamic amplification of waves in the ice which can give rise to stresses close to three times normal.
- Test the ice first with minimum criteria for breakthrough ie 24 cm for 1000kg vehicle.
- Mark parking and roadway with flag sticks in augured holes. Post warnings and statements of fines for leaving the authorized areas. Recommend that larger vehicles avoid parking in spaces adjacent to other vehicles.
- Keep track of ice deflections in the parking area and adjacent to the bubble near the clustered barrels. Use standard survey equipment and techniques. Note that the survey equipment should be placed at least 100 ice thicknesses from the loading source. Deflection tests can also be run inside the bubble using a limited series of drilled test holes under the bleachers and near the rinks. The freeboard can be measured relative to fixed plate on the ice surface. If the freeboard approaches 10 mm then suspend use of the facility and do not resume use until freeboard again becomes 8-10 % of ice thickness. Flooding the ice can be disastrous. Temperature increase leads to a strength reduction and in combination to overburden loads this can lead to failure.

REFERENCES

- Ashton, G.D. 1986, **River and Lake Ice Engineering**. Water Resources Publications, Colorado U.S.
- Gold, L. 1971, "Use of Ice covers for transportation". *Can Geotechnical Journal*, 8, pp.170-181.
- Jordaan I. 1994 ,Personal communication Eng 9051.
- Kerr, A. 1976 "The Bearing Capacity of Floating Ice Plates Subjected to Static or Quasi-Static Loads". *Journal of Glaciology*, Vol 17, No.76, 1976.
- Michel, B. 1978, *Ice Mechanics*. Le Presses De L'universite Laval, Quebec Can
- Nevel, D. Assur, A. 1968, "Crowds on Ice". CRREL Technical Report 204.
- Sanderson, T. 1988, *Ice Mechanics - Risks to Offshore Structures*. Graham and Trotham Ltd London, UK.
- Sinha, N, 1983, "Creep Model of Ice for Monotonically Increasing Stress". *Cold Regions Science and Technology* *, 25-33.
- Sinha, N. 1992, "Winterlude 1986 - Dows Lake Ice Loading Test". ASME, Vol IV, Polar Technology, 1992.

LEGEND

- . = elastic strain
- = delayed elastic strain
- o = secondary creep strain
- = total strain

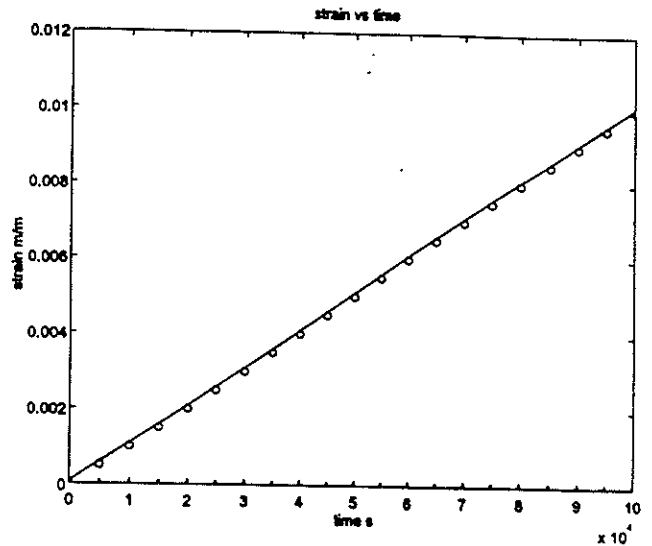
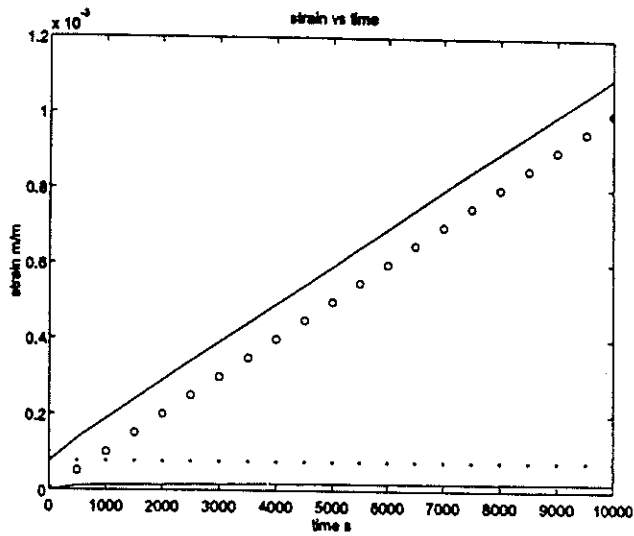
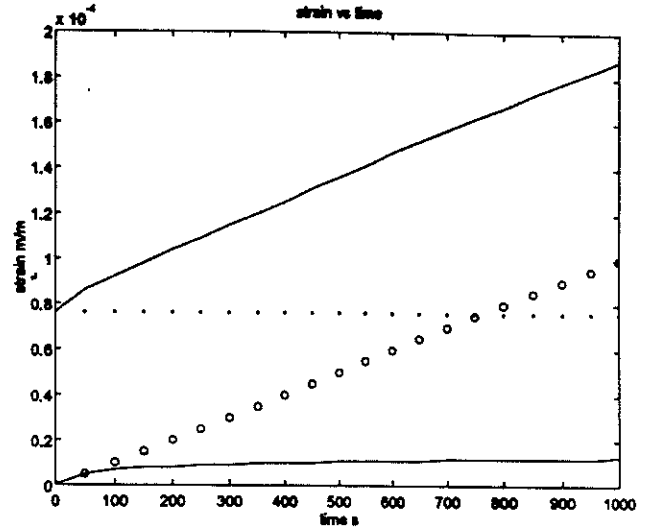
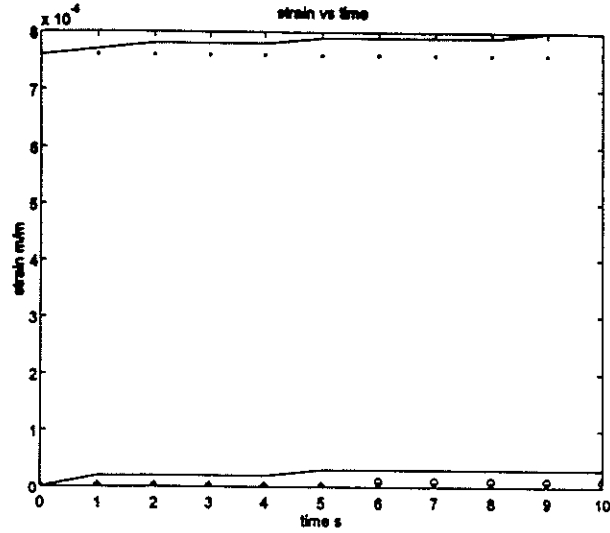


Figure 1 Uniaxial Strain vs time for constant stress. Duration is (a) 10 seconds, (b) 1000 s, (c) 10000 s, (d) 100000 s. $\sigma = 700 \text{ KPa}$, grain diameter = 2.5 μm .

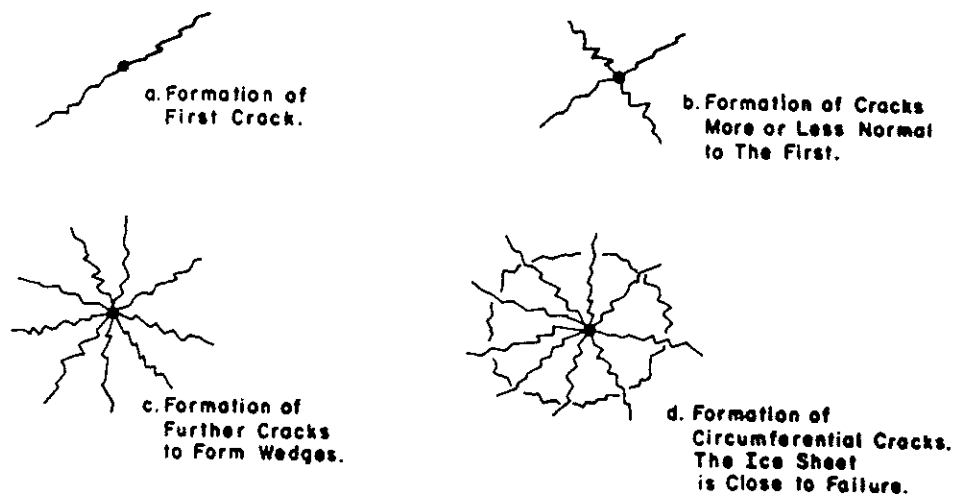


Figure 2 Stages in the failure of an ice sheet (Ashton, 1986).

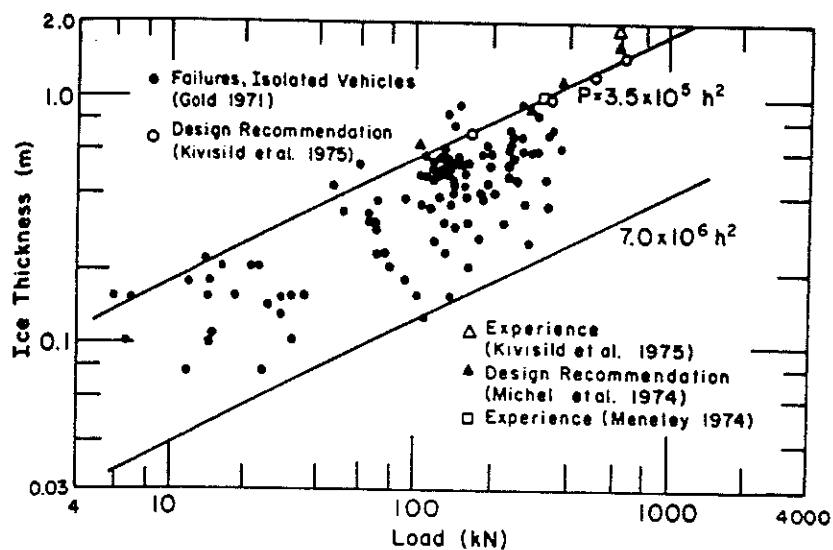


Figure 3 Failure loads reported during operations on ice (After Gold, 1971).

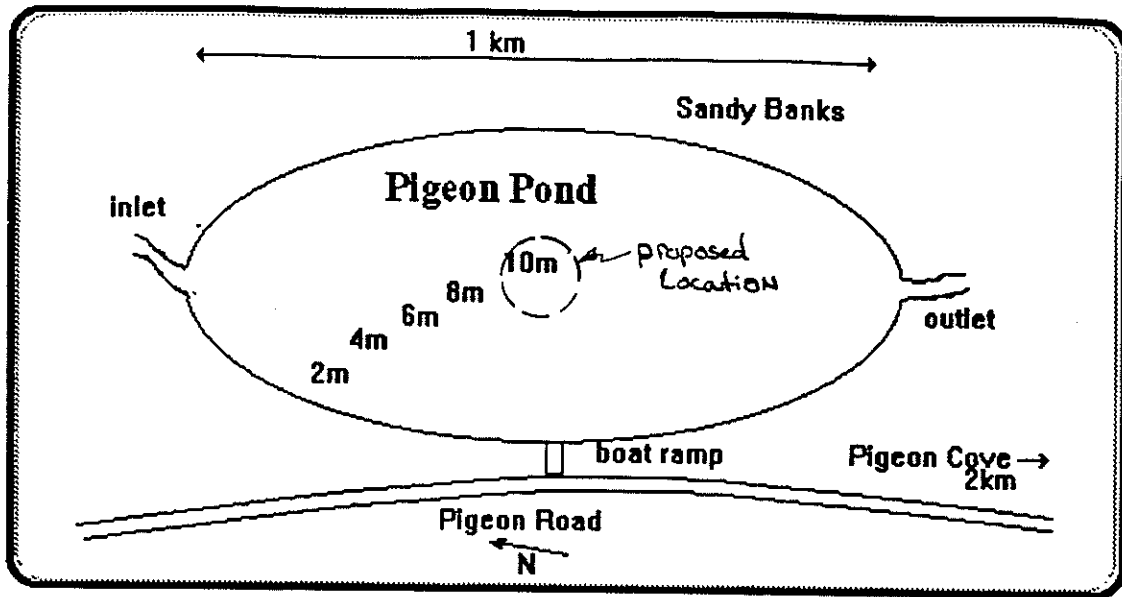


Figure 4 Case study freshwater pond in Northern Newfoundland.

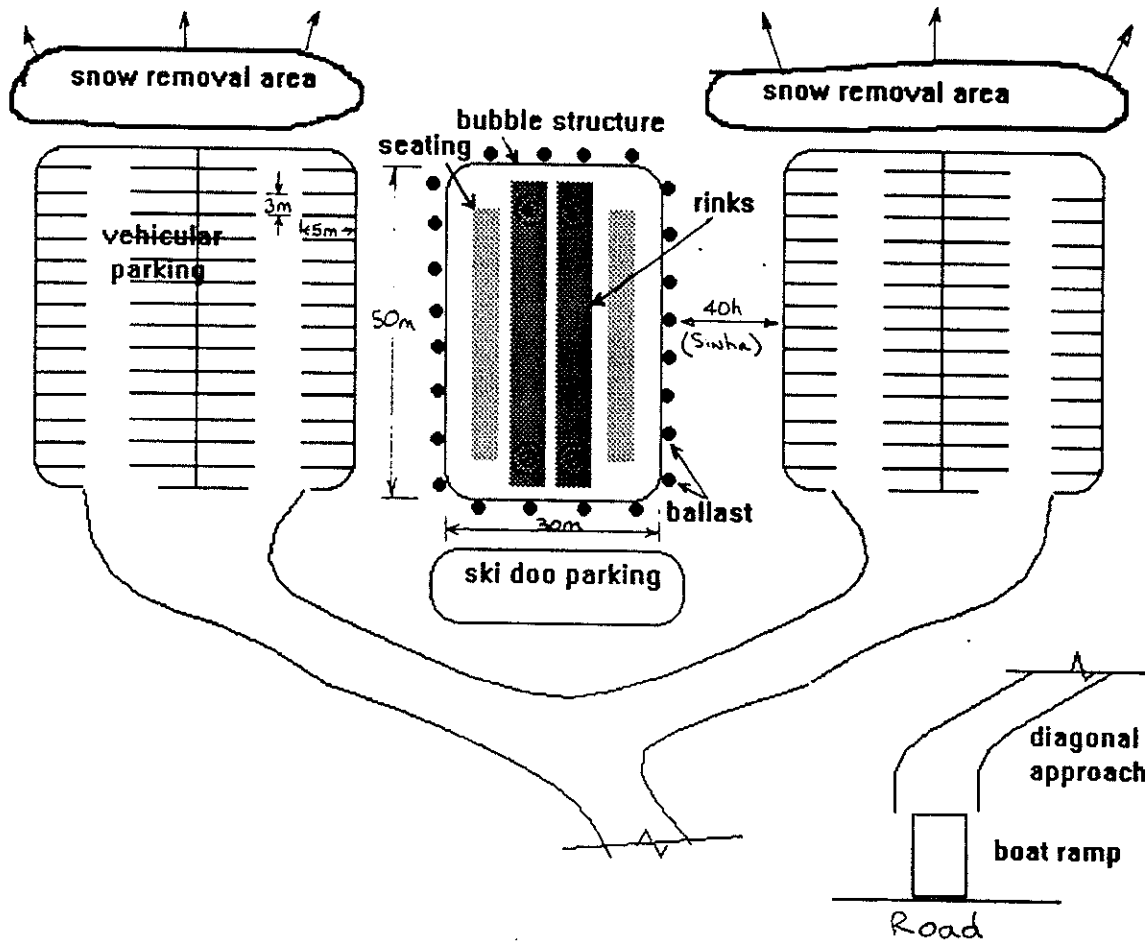


Figure 5 Preliminary layout of recreational curling facility on Pigeon Pond.

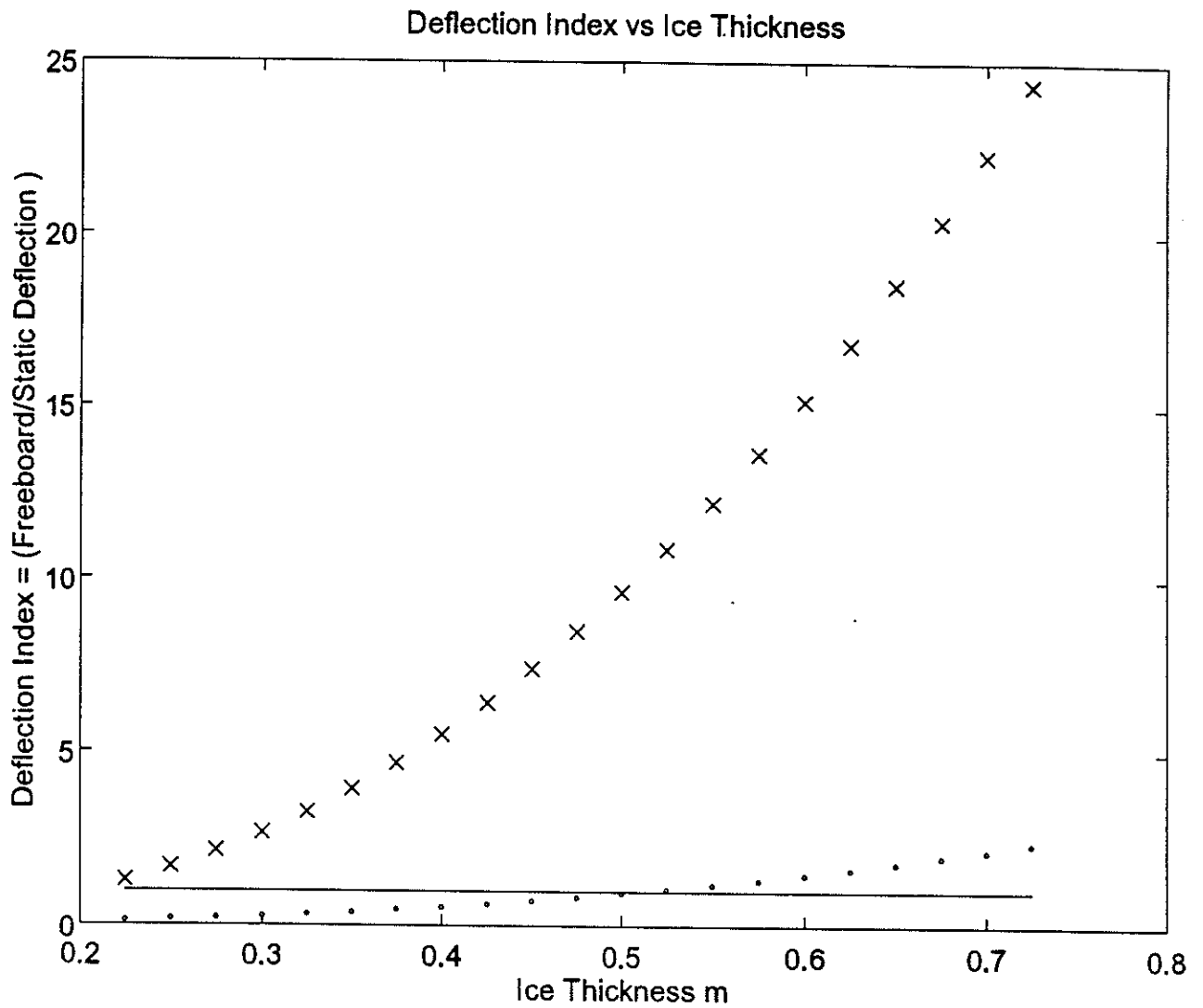


Figure 6 Deflection index (unloaded freeboard height / deflection) for given loading conditions for "x" Pigeon Pond ballast, and, "o" for Sinha (1992) Dows lake. Dows Lake ice thickness was .65m and Pigeon Pond criteria suggests >.42m required.

	$P_f(0)$		$P_f(t)$		
	<i>Based on elasticity analyses</i>		<i>Based on plasticity analyses</i>		
Analogy method for determination of P_a	Determination of P_{cr} based on elastic theory of plates and criterion $\sigma_{max} = \sigma_f$. Then correlation of P_{cr} and $P_f(0)$	Direct determination of $P_f(0)$ by analyzing the cracked plate. Use of elastic theory and criterion $\sigma_{max} = \sigma_f$	Determination of $P_f(0)$ using yield-line theory	Determination of $P_f(0)$ using limit-load theory	Use of a visco-elastic theory in conjunction with a failure criterion

Table 1 Procedures for determining failure load P_f and failure load for given storage time t_f . (After Kerr, 1976).

Load Source	Temperature	Load Condition	Load Quantity	Load Distribution	Requirements	Formulas	Results
Parking	T < -2 C	Stationary w/ Partial Plastic.	3000 kg	Concentrated	Check Deflection and Load capacity	$h=(P/3.5 \times 10^4)^{.5}$ $w=1.8(PI^2/8D)$	h = .3 m
			1500 kg				-
Roadway	T < -2 C	Moving/ Fully Elastic	3000 kg	Concentrated / Moving	Check Dynamic Load Capacity and Speeds	$h=(P/2.0 \times 10^4)^{.5}$	h = .39 m
			1500 kg				10 < v < 20 kph
Railroad	T < -2 C	Permanent w/ Full Plastic.	3500 kg / 7m	Concentrated/ Uniform	Check Load Capacity and long Term Deflection Maximize Safety Criterion Here	$h=(P/2.0 \times 10^4)^{.5}$ $w=1.8(PI^2/8D)$ (plus monitoring)	h = .42 m
			1800 kg / 9 m ²				w < 1/3 Freeb.
Seating/Spectators	-3 < T < 4 C	Stationary w/ Partial Plastic.	2.0 kn/m ² 400 kg / 9 m ²	Uniform	Maximize Safety Criterion Here	$h=(P/2.0 \times 10^4)^{.5}$ $w=1.8(PI^2/8D)$	= .3 m (3x3m)
Rink Surface	-3 < T < 4 C	Permanent w/ Full Plastification	0.4 Kn/m ² 400 kg / 9 m ²	Uniform	Check Deflection Criterion	$w=1.8(PI^2/8D)$	-

Table 2 Loading conditions and recommendations for case study.