Chapter 3

Fusion of Ascending and Descending Pass Interferograms

The previous chapters have described SAR and DInSAR methods to derive general ground movement. It was seen that with satellite repeat-pass interferometry, topographical maps could be obtained. This leads to the discussion of the DInSAR technique and ways it can be used to obtain one dimensional displacement vectors of the imaged scene. This chapter will further dissect the DInSAR technique and develop a means to fuse more than one look direction.

Using DInSAR, if one look direction is used to obtain ground movement estimates, then only one dimension of movement can be derived. Other directions of movement must then be assumed from the topography. For example, moving slopes are generally assumed to be moving along the slope
direction. This would be a poor assumption if there are known “slumps” along the slope. Also, regions that are generally flat are assumed to have no lateral movement component such that the only movement is vertical (heave or subsidence). However, even for flat regions, the assumptions of zero lateral movement can produce significant subsidence measurement errors. This is true especially in the case of significant localized movement, whereby the sides of the moving region “cave in” towards a central maximum (negative sign convention used for subsidence) as shown in Figure 3-1.

![Slides caving in produce significant lateral movement](image)

**Figure 3-1**: Sliding from both sides to produce lateral movement. Note: Figure B-4 in Appendix B illustrates the 3-D perspective.

Consequently, there is a rationale to improve ground movement estimates by fusing data from different look directions to produce 2D and possibly 3D movement. The sections below illustrate a technique to fuse different look directions, and with the aid of least square techniques to produce an estimate of 3D movement.
3.1 Ascending and Descending Pass Geometry

As mentioned previously, space-borne SAR allows imaging of two look directions from ascending and descending passes. The fusion of two differential interferograms obtained from ascending and descending passes can be understood on the basis of their imaging geometry.

Figure 3-2 below provides an illustration of the orbital path of a typical polar orbiting satellite that allows two different look directions from ascending and descending passes.

![Descending and ascending orbits](image)

**Figure 3-2**: Descending and ascending orbits (RADARSAT International, 1996)

Most polar orbiting satellites look in a single direction, either the left or the right of their orbital path. For example RADARSAT-1 and ERS-1/2 are right looking satellites, such that their look direction is to the East on the ascending pass, and to the West on the descending pass. The upcoming RADARSAT-2 can rotate its
antenna to produce both a left and right looking direction. As Figure 3-2 illustrates, the orbital path is tilted slightly off the poles; for RADARSAT-1 and ERS-1/2 this tilt is roughly 8° from true North at the equator. As the satellites approach the poles, the map-projected orbit varies significantly from the 8° tilt. This is illustrated in Figure 3-3, which shows the map-projected coverage of RADARSAT-1.

![Figure 3-3: Coverage area for RADARSAT-1 for both ascending and descending passes. (RADARSAT International, 1996)](image)

Figure 3-3 shows gradual change in map projected tilt angle from the equator, and then an abrupt curve in tilt angle at the top and bottom of the map. Over a small segment of the orbit (one or two images) that is not near the poles, the map
projected orbit path can be approximated by a straight line with a fixed tilt angle.

A plan view of the look directions is illustrated in Figure 3-4 below.

![Diagram of satellite look direction](image)

**Figure 3-4:** Plan view of satellite look direction illustrated

Now, consider a detailed profile view of the satellite look direction for each pass in Figure 3-5 and Figure 3-6. This profile view geometry has been discussed previously (cf. Figure 2-10), and will allow the decomposition of the differential slant range change to an actual ground movement vector. Consider the case where a point P on the ground has moved during a repeat pass and this movement vector is denoted as:

\[ \mathbf{B} = \mathbf{L} + \mathbf{S}, \]

where \( \mathbf{B} \) is the movement of point \( P \) to \( P' \) from pass one to pass two, \( \mathbf{L} \) is the lateral component (East-West and North-South components) and \( \mathbf{S} \) is the subsidence component. Using Cartesian coordinates:
\[ L = EW_x \hat{x} + NS_y \hat{y} = \Delta x \hat{x} + \Delta y \hat{y}, \]  
(3-1)

and
\[ S = S_z \hat{z} = \Delta z \hat{z} ; \]  
(3-2)

thus, \( B \) can be represented as
\[ B = \Delta x \hat{x} + \Delta y \hat{y} + \Delta z \hat{z}, \]  
(3-3)

where \( \hat{x}, \hat{y} \) and \( \hat{z} \) denotes the East-West, North-South and Vertical direction in a right hand coordinate system with origin at \( P \).

Figure 3-5: Ascending pass. \( B \) is the actual movement vector from \( P \) to \( P' \), \( A \) is the measured component of \( B \) from ascending pass, \( S \) is subsidence component and \( L \) is lateral component.

As shown in Figures 3-5 and 3-6, \( A \) and \( D \) are obtained from ascending and descending differential interferogram, and are the measured components of the movement \( B \). Thus, \( A \) denotes the measured component of \( B \) along the SAR look
direction for an ascending pass as shown in Figure 3-5. Similarly D denotes the measured component of B along the SAR look direction for a descending pass as shown in Figure 3-6.

![Figure 3-6: Descending pass. B is the actual movement vector from P to P', D is the measured component of B from descending pass, S is subsidence component and L is lateral component.](image)

3.2 Ascending and Descending Pass Equations

The fact that the satellite measures a slant range change along the line of sight of the sensor is suggestive of the use of a circular coordinate system. This will conveniently allow the conversion to a Cartesian (map oriented) coordinate system to express ground movement in terms of vertical and lateral components. Consider now the relationship between the satellite coordinate system, consisting
of incidence angle $\vartheta$, orbit tilt angle $\varphi$ and slant range change ($D, A$, since these vectors are obtained from a differential interferogram) to the standard spherical coordinate system consisting of unit vectors $\langle \hat{\rho}, \hat{\theta}, \hat{\phi} \rangle$. As shown in Figure 3-7, the satellite incidence angle $\vartheta$ can be expressed in circular coordinates $\langle \rho, \theta, \phi \rangle$ as

$$\vartheta = \pi - \theta.$$  \hspace{1cm} (3-4)

![Figure 3-7: Relating look angle $\vartheta$ with respect to circular coordinate $\theta$](image)

The satellite orbit, presently expressed in terms of the tilt angle $\varphi$, can also be expressed in terms of the circular coordinates with the origin at the measured point on the ground shown in Figure 3-8. Since the geometry of the tilt angle is different for the ascending and descending passes, Figure 3-8 shows the two different cases relating $\varphi_a$ to $\phi_a$ on the ascending pass and $\varphi_d$ to $\phi_d$ on the
descending pass. Thus, the relation between orbit tilt to spherical coordinates becomes:

\[ \phi_a = \varphi_a, \text{ and} \]
\[ \phi_d = \pi - \varphi_d \]  

Finally, the slant range change vectors \( D \) and \( A \) are oriented in the \( \hat{\rho} \) direction. Therefore, it becomes possible to formulate the transformation of slant range change, look angle and look direction for two different look directions to standard spherical coordinates, replacing \( <\hat{A}, \varphi_a, \varphi> \) to corresponding \( <\rho_a, \theta_a, \varphi_a> \) for ascending pass geometry, and \( <\hat{D}, \varphi_d, \varphi_d> \) to \( <\rho_d, \theta_d, \varphi_d> \) for descending pass geometry.

**Figure 3-8:** Relating satellite co-ordinate system to geometrical coordinate system, where \( \varphi_a \) and \( \varphi_d \) are the angles relating to inclination of satellite trajectory with respect to geographical North-South for ascending and descending passes.
The final transformation from the spherical to map projected Cartesian (EW, NS and Subsidence) system is achieved with a standard dot product table (Table 3-9).

<table>
<thead>
<tr>
<th>.</th>
<th>$\hat{\rho}$</th>
<th>$\hat{\theta}$</th>
<th>$\hat{\phi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{x}$</td>
<td>$\sin \theta \cos \phi$</td>
<td>$\cos \theta \cos \phi$</td>
<td>$-\sin \phi$</td>
</tr>
<tr>
<td>$\hat{y}$</td>
<td>$\sin \theta \sin \phi$</td>
<td>$\cos \theta \sin \phi$</td>
<td>$\cos \phi$</td>
</tr>
<tr>
<td>$\hat{z}$</td>
<td>$\cos \theta$</td>
<td>$-\sin \theta$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

**Table 3-9:** Dot product table relating spherical coordinate system to Cartesian coordinate system

Using all of the previous relationships, equations can now be derived to allow the fusion of ascending and descending pass DInSAR data. For the descending pass geometry, the equations are

$$D = \Delta \rho_d \hat{\rho} = \Delta x \hat{x} + \Delta y \hat{y} + \Delta z \hat{z}$$  \hspace{1cm} (3-7)

$$B = \Delta x \hat{x} + \Delta y \hat{y} + \Delta z \hat{z}$$  \hspace{1cm} (3-8)

where,

$$\Delta x = \Delta \rho_d \sin (\theta_d) \cos (\phi_d)$$  \hspace{1cm} (3-9)

$$\Delta y = \Delta \rho_d \sin (\theta_d) \sin (\phi_d)$$  \hspace{1cm} (3-10)

$$\Delta z = \Delta \rho_d \cos (\theta_d),$$  \hspace{1cm} (3-11)

come directly from the dot product table of Figure 3-9 and $\Delta \rho_d$ is the measured slant range change in the descending direction. Hence the dot product of $B$ and $D$ is given by:

$$B \cdot D = \Delta x \cdot \Delta x + \Delta y \cdot \Delta y + \Delta z \cdot \Delta z$$  \hspace{1cm} (3-12)

$$B \cdot D = \Delta \rho_d \{ - \sin (\gamma_d) \cos (\phi_d) \Delta x + \sin (\gamma_d) \sin (\phi_d) \Delta y + \cos (\gamma_d) \Delta z \}$$  \hspace{1cm} (3-13)
Now, to get the component of $\mathbf{B}$ in the $\mathbf{D}$ direction, the dot product is divided by $|\mathbf{D}|$ as follows:

$$\frac{\mathbf{B} \cdot \mathbf{D}}{|\mathbf{D}|} = \mathbf{B} \cdot \hat{\mathbf{D}},$$

(3-14)

where,

$$|\mathbf{D}| = \Delta \rho_d$$

(3-15)

Expanding the above equation gives:

$$\mathbf{B} \cdot \hat{\mathbf{D}} = -\sin(\hat{\vartheta}_d \Delta) \cos(\varphi_d \Delta) \Delta x + \sin(\hat{\varphi}_d \Delta) \sin(\varphi_d \Delta) \Delta y + \cos(\hat{\varphi}_d \Delta) \Delta z .$$

(3-16)

Recall now the elements of equation (3-16):

- $\rho_d$ is the slant range change (differential interferogram of descending pass);
- $\varphi_d$ is the projection of the satellite trajectory for descending pass;
- $\vartheta_d$ is the corresponding incidence angle of the descending pass;
- $\Delta x$ is the displacement along the $x$ axis;
- $\Delta y$ is the displacement along the $y$ axis; and
- $\Delta z$ is the displacement along the $z$ axis.

Similarly it is possible to derive the equation for the ascending pass geometry.

$$\mathbf{B} \cdot \hat{\mathbf{A}} = -\sin(\hat{\vartheta}_a \Delta) \cos(\varphi_a \Delta) \Delta x - \sin(\hat{\varphi}_a \Delta) \sin(\varphi_a \Delta) \Delta y + \cos(\hat{\varphi}_a \Delta) \Delta z ,$$

(3-17)

where in this case,

- $\varphi_a$ is the projection of the satellite trajectory for ascending pass
- $\vartheta_a$ is the corresponding incidence angle of the ascending pass.
Using equation (3-16) and (3-17) it is now possible to complete a relationship between components $\Delta x, \Delta y, \Delta z$ and slant range change measurements. The derived set of equations can be represented in a compact matrix form:

\[
\begin{bmatrix}
-\sin(\vartheta_d) \cos(\varphi_d) & \sin(\vartheta_d) \sin(\varphi_d) & \cos(\vartheta_d) \\
-\sin(\vartheta_a) \cos(\varphi_a) & -\sin(\vartheta_a) \sin(\varphi_a) & \cos(\vartheta_a)
\end{bmatrix}
\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z
\end{bmatrix}
= 
\begin{bmatrix}
\Delta \rho_d \\
\Delta \rho_a
\end{bmatrix}.
\] (3-18)

Equation (3-16) and (3-17) can be independently used to extract any one of the three dimensional ground movement components, either by setting the other two components to be zero or by using prior information about their magnitude and direction. The system of equations (3-18) is under-determined with two equations and three unknowns. The incidence angles of the SAR, $\vartheta_a$ and $\vartheta_d$ are represented as constants over the entire scene and the representative value is at the center range pixel of the SAR scene. The incidence angle can be calculated for each range pixel of the SAR scene with the knowledge of its value at the center. Though the change in incidence angle from one pixel to the other is very small over a given scene, this information can be utilized to construct an over-determined system of equations. The variations in $\varphi_a$ and $\varphi_d$ are negligible over a single scene and are often treated as constants.

To resolve the under-determined system, as a first approximation, one of the displacement variables can be assumed to be zero. For example, polar orbiting satellites have look directions that are predominantly in the East-West
direction. As a result, the slant range change is less sensitive to lateral movement in the North-South direction. Thus, a first approximation could use the assumption that lateral movement in the North-South direction (i.e. $\Delta y$) is zero.

### 3.3 Estimating 3-D Ground Movement

The synthesis equations that were derived for extracting ground movement from the two non-parallel passes can, in general, be used to derive two dimensional ground movement. In principle, only one component of the displacement vector can be obtained from a single interferometric pair of similar viewing geometry. To measure three components of displacement, one must have three sets of interferometric pairs, each of which have different look directions, unless additional information (e.g., from ground observations) are available to determine the full three-dimensional displacement field. To extract the third component of movement, assumptions have to be made in the absence of a third differential interferogram with unique look direction. With satellite interferometry, it is not possible to obtain this third unique pair. However, under certain conditions it may be possible to estimate three dimensional ground movement using least squares techniques. These conditions include regions experiencing well-behaved ground movement, in which the movement is generally homogeneous over many resolution cells of the SAR sensor.
With reference to Figure 3-10, consider a small area on the ground viewed by a varying incidence angle $\vartheta$ to $\vartheta + \Delta \vartheta$ over which the surface change is a small-scale coherent change common to several adjacent pixels. In other words, the deformation is well behaved with no discontinuities. With this assumption, the under-determined system in equation (3-18) can be restructured into an over-determined system for pixels within the region bounded by $(\vartheta, \vartheta + \Delta \vartheta)$. Consider $n$ pixels in that neighborhood where the position of the radar scatter has not changed substantially; however, the ensemble of the scatter has moved up, down, or sideways in some correlated fashion. This implies that an area on the ground experienced homogenous movement and can be collectively grouped by those $n$ pixels.

**Figure 3-10:** Bounded region of $n$ pixels that coherently move together.
For the $i^{th}$ pixel in that $n$ pixel region (Figure 3-10), the 3-D movement $\Delta x_i, \Delta y_i$ and $\Delta z_i$ can now be estimated. The conversion matrix for each pixel $i$ is given as

$$
\begin{bmatrix}
-\sin(\varphi_d) & \sin(\varphi_d) & \cos(\varphi_d) \\
-\cos(\varphi_a) & -\sin(\varphi_a) & \cos(\varphi_a)
\end{bmatrix}
\begin{bmatrix}
\Delta x_i \\
\Delta y_i \\
\Delta z_i
\end{bmatrix}
= 
\begin{bmatrix}
\Delta \rho_{\varphi_d} \\
\Delta \rho_{\varphi_a}
\end{bmatrix},
$$

(3-19)

The least squares solution for the system of equations in (3-19) can be represented as $MX = b$ where $M$ is the system matrix,

$$
X = \begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z
\end{bmatrix}
$$

and $b = \begin{bmatrix}
\Delta \rho_d \\
\Delta \rho_a
\end{bmatrix}$.

The least squares solution of $X$ is such that the size of the residual vector is minimal. The over-determined system of equations permits a solution for each $\Delta x_i, \Delta y_i$ and $\Delta z_i$; however, as discussed in Section 3.2, meaningful measurements might only be obtained from $\Delta x_i$ and $\Delta z_i$. The reason is due to the imaging geometry of the SAR satellite (Figure 3-10) whereby the slant range change (SRC) is relatively insensitive to changes in the $y$ direction compared to $x$ and $z$ directions. As a consequence, the measurement error of the SAR sensor may be greater than that of the measured $\Delta y$ (North-South) movement, except in the case of substantial lateral movement. Averaging many interferograms might
reduce the measurement errors sufficiently to allow reasonable estimates in the y direction.

The grid for the over determined system of equations with overlapping ascending and descending pass images is illustrated in Figure 3-11. The variation of $\delta u_i$ and $\delta d_i$ over a small area on the ground is insignificant. Therefore a relatively large area has to be chosen to provide a meaningful result from the least squares estimation. The rationale for this is based on the convergence of a least squares solution. However, it also reduces the likelihood of having a region of homogeneous ground movement with no discontinuities. To mitigate these potential problems, an optimization routine can be employed to determine the region of most suitable size. This can be achieved by varying the size of $n$ (number of pixel as a variable) pixel neighborhood as shown in Figure 3-11 to estimate subsidence movement using any one of the two differential interferogram (either ascending or descending) until a satisfactory level of correlation is achieved when compared with data obtained from the global positioning systems (GPS). By doing this, a number of homogeneous ground movement pixels can be estimated for the region under investigation. This number can then be used for the least squares solution to estimate ground movement in 3-D.
Figure 3-11: Ascending and descending pass grid with varying incidence angle. The $n$ neighborhood identifies region of continuous displacement field used for the least-squares solution.

In summary, this chapter has derived a technique to estimate 3D ground movement from DInSAR by fusing ascending and descending pass images. It has also illustrated the difficulty in extracting 3D ground movement vectors from only two look directions. If any of the movement components are known a priori, then the other two components can be easily estimated except for the lack of sensitivity in the north-south direction. With a homogeneous ground movement assumption, mathematical modeling techniques were derived to estimate the three unknowns. The verification of the proposed method is explored in the following chapter, which illustrates and explains the rationale of
each verification step and resolves issues with the results as encountered. In this chapter a test region is used to illustrate the validity of the movement measurement technique by comparing DInSAR derived movements with in-situ measurements.