Vertical Curves (Chapter 25)

- Curves are needed to provide smooth transitions between straight segments (tangent) of grade lines for highways and railroads.
- In addition to horizontal curves that go to the right or left, roads also have vertical curves that go up or down.
- These curves are used to join tangents (eg: tangent 1, 2 and 3 below ) in order to provide a gradual change in grade from the initial (back) tangent to the grade of the second (forward) tangent.

Vertical curves at the top of a hill are called **crest curves** and vertical curves at the bottom of a hill or dip are called **sag curves**.
Factors to be Considered (section 25.1)

There are several factors that must be taken into account when designing a grade line of tangents and curves on any highway or railroad projects. They include:

- Providing a good fit with the existing ground profile, thereby minimizing depths of cuts and fills.
- Balancing the volume of cut materials against fill.
- Maintaining adequate drainage.
- Not exceeding maximum specified grades \( g \) and meeting fixed elevations such as intersections with other roads.
- In addition, the curves must be designed to
  - fit the grade lines they connect
  - have lengths sufficient to meet specifications covering a maximum rate of change of grade (which affects the comfort of vehicle occupants)
  - provide sufficient sight distance for safe vehicle operation.

\[ g = 4\% \]
Vertical Curve Geometry (section 25.2)

Parabolas provide a constant rate of change of grade, they are ideal and almost always applied for vertical alignments used by vehicular traffic.

The general mathematical expression of a parabola:

\[ Y_p = a + bX_p + cX_p^2 \]  \hspace{1cm} (1)

- \( Y_p \) = the ordinate at any point \( p \) of the parabola at a distance \( X_p \) from the origin of the curve
- \( a \) = the ordinate at the beginning of the curve (\( X = 0 \))
- \( b \) = the slope of the tangent to the curve (\( X = 0 \))
- \( bX_p \) = the change in ordinate along the tangent over distance \( X_p \)
- \( cX_p^2 \) = the parabola’s departure from the tangent (tangent offset) in distance \( X_p \)
Elements of Vertical Curve

Equal Tangent Vertical Parabolic curve (section 25.3)

Terms used by surveyors and Engineers:

BVC = beginning of vertical curve  OR
VPC = vertical point of curvature
V = the vertex, often called VPI
VPI = vertical point of intersections
EVC = end of vertical curve  OR
VPT = vertical point of tangency

g_1 = grade of the back tangent (%)
g_2 = grade of the forward tangent (%)
L = horizontal distance (BVC to EVC)

An **equal tangent vertical parabolic curve** means the vertex occurs at a distance \( X = \frac{L}{2} \) from the BVC.

Elements of Vertical Curve

Equal Tangent Vertical Parabolic curve (section 25.3)

Using surveying terminology eq. 1 becomes:

\[ Y = Y_{BVC} + g_1 X + cX^2 \]  \hspace{1cm} (2)

To express the constant \( c \):

\[ g_1 L - (-cL^2) = g_1 \frac{L}{2} - (-g_2 \frac{L}{2}) \]

\[ \Rightarrow cL^2 = g_1 \frac{L}{2} + g_2 \frac{L}{2} - g_1 L \]

Solving the constant \( c \) gives:

\[ c = \frac{g_2 - g_1}{2L} \]  \hspace{1cm} (3)

Substituting \( c \) in to eq 2 gives:

\[ Y = Y_{BVC} + g_1 X + \left( \frac{g_2 - g_1}{2L} \right) X^2 \]
Elements of Vertical Curve

*The rate of change of grade, r*, for an equal tangent parabolic curve equals the total grade change from BVC to EVC divided by length L (on stations for the English system, or L/100 or 1/10th stations for metric units), over which the change occur.

\[ r = \frac{g_2 - g_1}{L} \]

This is the same as the second derivative of the vertical curve equation.

The slope of the curve at any point:

\[ \frac{dY}{dX} = g_1 + 2cX \]

*The rate of change grade (r)* is given by the second derivative:

\[ \frac{d^2Y}{dX^2} = r = 2c = 2 \frac{g_2 - g_1}{2L} = \frac{g_2 - g_1}{L} \]

The value of r (*which is negative for a crest curve and positive for sag curve*) is an important design parameter because it controls the rate of curvature and hence rider comfort.

Substituting \( r \) in to eq 3 gives:

\[ Y = Y_{BVC} + g_1X + \left( \frac{r}{2} \right)X^2 \]
**Example (25.2, text book)**

**Given:**
- \( g_1 = -3.629\% \), \( g_2 = 0.151\% \)
- VPI station = 5+265.000 m
- \( E_{VPI} = 350.520 \) m
- \( L = 240 \) m
- Increments = 40 m (staking)

**Soln:**
- \( r = \frac{g_2 - g_1}{L} = \frac{0.151 + 3.629}{2.4} = 1.575 \)
- **Stationing**
  - VPI Station = 5+265
  - \(-L/2 = 120\)
  - BVC Station = 5+145
  - \(+L = 240\)
  - EVC Station = 5+385

**Soln:**
- \( E_{BVC} = E_{VPI} + g_1 \cdot L/2 \)
  - \( Y_{BVC} = 350.520 + 3.629 \cdot 120/100 \)
  - \( Y = Y_{BVC} + g_1X + \left( \frac{r}{2} \right)X^2 \)

<table>
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<tr>
<th>Station</th>
<th>( X(m/100) )</th>
<th>( g_1X )</th>
<th>( rX^2/2 )</th>
<th>Curve Elevation (m)</th>
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<tr>
<td>5+145 (BVC)</td>
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<td>0.000</td>
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<td>3.640</td>
<td>350.713</td>
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<tr>
<td>5+385 (EVC)</td>
<td>2.400</td>
<td>-8.710</td>
<td>4.536</td>
<td>350.701</td>
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</tbody>
</table>
Sight Distance (section 25.11)

The vertical alignments of highways should provide ample sight distance for safe vehicular operation. Two types of sight distances are involved:

- **stopping sight distance** (the distance required for a given design speed to safely stop a vehicle thus avoiding a collision with an unexpected stationary object in the roadway a head).

- **passing sight distance** (the length of roadway that the driver of the passing vehicle must be able to see initially, in order to make a passing maneuver safely).

The American Association of State Highway and Transportation Officials (AASHTO) has recommended minimum sight distance for both stopping and passing for various design speed (Table 25.4, textbook).