Problem set 2

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Important: For all these problems, do not worry about efficiency. We will explore efficient approaches later. For now I want to focus on looking at problems as instances of more general sub problems that can be broken down.

Q0 We need to break a sequence of items of various weights into roughly equally weighted segments. The problem is represented as a sequence of n positive real weights $w = [w_0, w_1, .., w_{n-1}]$, a positive real number x, and a positive integer p. The goal is to break w into p segments such that no segment weighs more than x. For each segment, there is a penalty of the cube of the difference between the weight of the segment and x. If we represent the segments by an array of $p+1$ numbers $k(0)$, $k(1)$, ... , $k(p)$ where

$$0 = k(0) \le k(1) \le \cdots k(p-1) \le k(p) = n,$$

then the segments are $[w_{k(0)}, w_1, .., w_{k(1)-1}]$, $[w_{k(1)}, w_{k(1)+1}, .., w_{k(2)-1}]$, ... and $[w_{k(p-1)}, w_{k(p-1)+1}, .., w_{k(p)-1}]$. An acceptable solution has

$$w_{k(i)} + w_{k(i)+1} + \cdots + w_{k(i+1)-1} \le x$$

for each $i \in \{0, ..p\}$. An optimal solution minimizes the total penalty, which is

$$\sum_{i \in \{0,..p\}} \left(x - w_{k(i)} - w_{k(i)+1} - \cdots - w_{k(i+1)-1}\right)^3 .$$

Design a function to compute the cost (i.e., total penalty) of an optimal solution for inputs w, x, and p. Your function should return ∞ if there is no solution. Hint: For a given w, x, and p, you can define subproblems defined by integers i and q, such that $0 \le i \le n$ and $0 \le q \le p$; subproblem (i, q) is to find the cost of an optimal way to split the first i items of w into q segments, each of which weighs less than x. The original problem is just the subproblem such that $i = n$ and $q = p$.

- Specify using pre- and postconditions
- Write the function body

Q1 Suppose we represent a complex project by a simple directed acyclic graph $G = (V, E)$. Each vertex represents a milestone (including a start and finish milestone), while each edge represents a task to be done. Each task is associated with a time, which is the time it will take to complete. You need to find the total time of the longest path from the start to the finish.

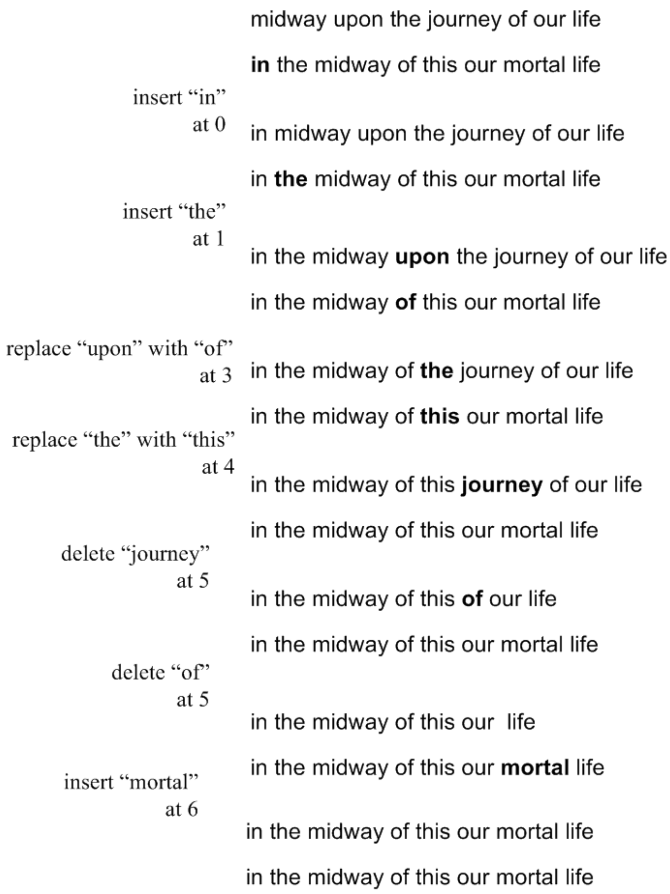
- Specify subproblems using pre- and postconditions.
- Write the function body.

Q2 Given a set of items S, produce the set of all permutations of items in S. Try to solve this problem in two different ways.

Q3 Given two sequences, how many operations are needed to transform one into the other Each operation is one of

- Delete an item
- Add an item
- Replace one item with another

Example: This edit sequence has 7 operations. Is this minimal?



- Identify the subproblems.
- Specify with pre- and postconditions.
- Design the body