Assignment 1

Advanced Computing Concepts for Engineering

Due January 18, 2018

Note that the work that you turn in for this assignment must represent your individual effort. You are welcome to help your fellow students to understand the material of the course and the meaning of the assignment questions, however, the answer that you submit must be created by you alone.

Please consider preparing your assignment with a typesetting program such as TeX, LaTeX, LyX, Scientific Word, or MS Word.

Q0 Signatures and behaviours

Suppose that we have a system with one input of type $\{0, 1\}$ and one output of type $\{0, 1, 2\}$.

(a) What would be a signature for the system.

(b) List all behaviours that belong to that signature.

(c) Which of these behaviours are accepted by a specification that requires that the output be larger than the input.

Q1 Counting

Suppose that we have a signature $\{\text{“}x\text{”} \mapsto S, \text{“}y'\text{”} \mapsto T\}$.

(a) In terms of $|S|$ and $|T|$ (which you may assume are natural numbers), how many behaviours belong to the signature?

(b) How many specifications are there with this signature?

Q2 Writing specifications.

Suppose n is some fixed natural number greater than 0. Then $\mathbb{Z}^n = \left(\{0, ..n\} \stackrel{\text{tot}}{\to} \mathbb{Z}\right)$ is the set of all sequences of integers of length n. Let

$$\Sigma = \{\text{“}x\text{”} \mapsto \mathbb{Z}, \text{“}a\text{”} \mapsto \mathbb{Z}^n, \text{“}x'\text{”} \mapsto \mathbb{Z}, \text{“}a'\text{”} \mapsto \mathbb{Z}^n\}$$

Where x and x' represent, respectively, the initial and final values of a program variable x, and a and a' represent, respectively, the initial and final values of a program variable a.

Using angle-bracket notation, give a mathematical version of the following specifications:

(a) The final value of x is the sum of the items in the initial value of a.

(b) The every item of the final value of a is the same as the initial value of x. The final value of x is the same as its initial value.

(c) If the initial value of x is in the set $\{0, ..n\}$, the final value of a is the same as its initial value except that item x is set to 0. Otherwise, if the initial value of x is not in the set $\{0, ..n\}$, it does not matter what the final values are.

(d) The initial value of x must be the same as the first item of the initial value of a.

Q3 Refinement

(a) Make a table of all (9) behaviours belonging to $\Sigma \dagger \Sigma$ where[1]

$$\Sigma = \{\text{“}x\text{”} \mapsto \{1, 2, 3\}\}$$

For each behaviour, indicate whether it is accepted (✓) or rejected (×) by each of the following specifications (on $\Sigma \dagger \Sigma$)

$$\begin{aligned} a &= \langle x < 3 \Rightarrow x' < 3 \rangle_{\Sigma\dagger\Sigma} \\ b &= \langle x < 2 \Rightarrow x' < 3 \rangle_{\Sigma\dagger\Sigma} \\ c &= \langle x \leq 3 \Rightarrow x' < 3 \rangle_{\Sigma\dagger\Sigma} \\ e &= \langle x < 3 \Rightarrow x' \leq 3 \rangle_{\Sigma\dagger\Sigma} \\ f &= \langle x < 3 \Rightarrow x' < 2 \rangle_{\Sigma\dagger\Sigma} \\ g &= \langle x < 3 \wedge x' < 3 \rangle_{\Sigma\dagger\Sigma} \\ \textbf{magic} &= \langle \mathfrak{false} \rangle_{\Sigma\dagger\Sigma} \\ \textbf{abort} &= \langle \mathfrak{true} \rangle_{\Sigma\dagger\Sigma} \end{aligned}$$

(b) What are the refinement relations between the specifications in part (a). Illustrate these relationships with a Hasse diagram.[2]

Q4 Specification

Write formal specifications for the following informal specifications. In each case the signature is $\{\text{“}x\text{”} \mapsto \mathbb{N}, \text{“}x'\text{”} \mapsto \mathbb{N}\}$. Use angle bracket notation.

(a) x' is bigger than x.

[1] $\Sigma \dagger \Sigma$ is $\{\text{“}x\text{”} \mapsto \{1, 2, 3\}, \text{“}x'\text{”} \mapsto \{1, 2, 3\}\}$

[2] You can look up Hasse diagrams on Wikipedia.

(b) x' exactly twice as big as x if x is smaller than 100; otherwise it doesn't matter what x' is.

(c) x' exactly twice as big as x if x is smaller than 100; otherwise x' is 200.

(d) x is at least 1 and x' is smaller than x.

Q5 Determinism and Implementability

Looking back at the specifications in questions Q3 and Q4, which are deterministic? Which are implementable?