Assignment 2

Advanced Computing Concepts for Engineering

Due January 25, 2018

Note that the work that you turn in for this assignment must represent your individual effort. You are welcome to help your fellow students to understand the material of the course and the meaning of the assignment questions, however, the answer that you submit must be created by you alone.

Q0

(a) Prove that if g is implementable and $f \sqsubseteq g$, then f is implementable.

(b) Prove that if $g \sqsubseteq h$ then $f; g \sqsubseteq f; h$.

Q1 If we use the erasure law to show $\langle x > 0 \vee x' < 0 \vee x = 0 \rangle \sqsubseteq \textbf{skip}$, what expression must be shown to be universally true.

Q2 Given $\Sigma = \{\text{“}x\text{”} \mapsto \mathbb{R}, \text{“}y\text{”} \mapsto \mathbb{R}\}$, implement the following specification using a sequence of nonparallel (single-variable) assignments without using any additional variables.

$$\langle x' = y \wedge y' = x \rangle$$

Use forward substitution and erasure laws. [Hint use arithmetic operations.] As you do this, state which expressions need to be shown universally true.

Q3 Given $\Sigma = \{\text{“}x\text{”} \mapsto \mathbb{R}, \text{“}y\text{”} \mapsto \mathbb{R}, \text{“}z\text{”} \mapsto \mathbb{R}\}$, use the alternation law (and others) to implement

$$\langle (x < z \wedge y' = -1) \vee (x > z \wedge y' = 1) \vee (x = z \wedge y' = 0) \rangle$$

Show and justify each refinement.

Q4 Binary search. Given a constant $N > 0$ and a constant function $C : \{0, ..N\} \xrightarrow{\text{tot}} \mathbb{N}$, that is sorted (nondecreasing). We want to see whether there is an item of C that equals x. I'll use the notation $C\{p, ..r\}$ for the set of items with indecies in set $\{p, ..r\}$, i.e., $C\{p, ..r\} = \{k \in \{p, ..r\} \cdot C(k)\}$. Suppose we have variables p, q, r of type $\mathbb{N}$. Our specification is $f = \langle (x \in C\{0, ..N\}) = (x = C(p')) \rangle$. We can 'generalize' f as

$$g = \langle 0 \leq p < r \leq N \Rightarrow (x \in C\{p, ..r\}) = (x = C(p')) \rangle$$

In the following be sure to give a justification for each refinement.

(a) Find an initialization command i such that $f \sqsubseteq i; g$.

(b) Implement g recursively using an alternation. Try to ensure that each iteration reduces $r - p$ to roughly half its value. [Hint: Because C is nondecreasing: If $0 \leq p < q < r \leq N$ and $C(q) > x$, $(x \in C\{p, ..r\}) = (x \in C\{p, ..q\})$ .And

if $C(q) \leq x$ then $(x \in C\{p, ..r\}) = (x \in C\{q, ..r\})$.] Be sure to justify each step of your derivation.

(c) Apply (the incomplete version of) the while law to implement g with a while-command. However you should still informally check that your loop will terminate, so state a bound expression for the loop.

Q5 Russian peasant multiplication

Suppose x, y, and z are natural number variables. We want to implement: $f = \langle z' = x \times y \rangle$ without using a multiplication.

In the following be sure to give a justification for each refinement.

(a) Find a 'generalization' g that will work with the initialization command $z := 0$. I.e. we want $f \sqsubseteq z := 0; g$.

(b) Implement g recursively using an alternation. We would like the time to be proportional to the $\log_2 y$. You will find the following identities useful: $x \times y = x + x \times (y - 1)$ if $y > 0$. And $x \times y = 2 \times x \times y/2$ if y is even. Be sure to justify each step of your derivation.

(c) Apply (the incomplete version of) the while law to implement g with a while-command. However you should still informally check that your loop will terminate. State a bound for your loop.