Midterm Solution

Advanced Computing Concepts for Engineering

2015

Total marks: **52**
Name:
Student #:
Q0 [4] Suppose that a and b are functions from natural numbers (representing times) to booleans. Using quantifier notation and/or set notatio to express the following requirements.

- Any time a is true, b will be true at some later time.

  Solutions:

  $$\begin{aligned} \forall t &\in \mathbb{N} \cdot a(t) \Rightarrow (\exists u \in \mathbb{N} \cdot u > t \wedge b(u)) \\ \forall t &\in \mathbb{N} \cdot a(t) \Rightarrow (\exists u \in \mathbb{N} \cdot b(u + t + 1)) \end{aligned}$$

- Any time a is true, b will be true within 100 time units.

  Solutions:

  $$\begin{aligned} \forall t &\in \mathbb{N} \cdot a(t) \Rightarrow (\exists u \in \mathbb{N} \cdot t \le u < 100 \wedge b(u)) \\ \forall t &\in \mathbb{N} \cdot a(t) \Rightarrow (\exists u \in \{0, ..100\} \cdot b(u + t)) \end{aligned}$$

  There is room for interpretation as to what "within 100 time units". Such ambiguity of natural language is one reason stating things formally can be very useful.

Q1 [4]. An inversion is an item in an array whose value is greater than the next item. Give a specification for the following problem: Given an array a, of size N, set k to the number of inversions in a.

Solution:

$$\langle k' = |\{i \in \{0, ..N - 1\} \mid a(i) > a(i + 1)\}|\rangle$$

Q2 [14] Suppose a is an integer function with domain $\{0,..,n\}$ where $n \geq 1$. We need to find a place in the array where $a(k) > a(k+1)$.You may assume that $a(0) > a(n)$.

$$f = \langle a(k') > a(k'+1)\rangle$$

(a)[6] Give a loop invariant $\mathcal{I}$.

Solution: Of course there is no one right answer. What is important is that invariant, code, and bound work together. Here is the invariant I chose to use

$$\mathcal{I} \text{ is } 0 \leq k < q \leq n \wedge a(k) > a(q)$$

Other invariants that could be used include

$$0 \leq k < n \wedge a(k) > a(n)$$

and

$$0 \leq k < n \wedge a(0) > a(k+1)$$

but these would require different answers to the other parts.

Marking notes: Very few students gave answers to this part that could lead to correct answers to the other parts. Thus I marked to question as a whole rather than by breaking it down into parts according to the marking scheme.

(b)[2] Let $g = (\langle\mathcal{I}\rangle \Rightarrow f)$ and $m = \langle a' = a \wedge \mathcal{I}'\rangle$ Give an initialization command that refines m.

Solution:

$$m \sqsubseteq k, q := 0, n$$

(c)[2] Give a guard expression $\mathcal{A}$ such that $\langle\neg\mathcal{A}\rangle \Rightarrow g \sqsubseteq \textbf{skip}$

Solution:

$$\mathcal{A} \text{ is } q \neq k+1$$

(d)[2] Give a loop body command that refines $h = \langle\mathcal{A} \wedge \mathcal{I} \Rightarrow \mathcal{I}' \wedge a' = a\rangle$ and decreases a bound.

Solution:

$$h \sqsubseteq m := \left\lfloor \frac{k+q}{2} \right\rfloor ; \textbf{if } a(m) > a(q) \textbf{ then } k := m \textbf{ else } q := m$$

(e)[2] Give a bound expression with which your loop can be shown to terminate.

$$q - k$$

Q3 [6]

(a) Let $f = \langle x > 0 \Rightarrow 0 \le x' < x \rangle$. Give an assignment command that refines f.

Solution: Many solutions are possible. Here is one

$$f \sqsubseteq x := x/2$$

Another is $x := 0$. If we assume x is an integer variable, then $x := x - 1$ would also work.

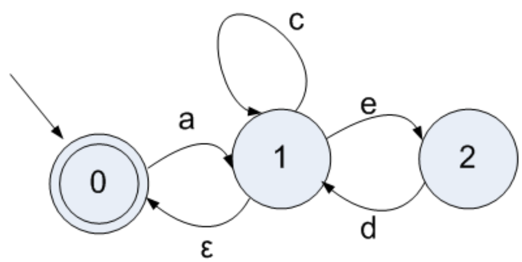
(b) Use the erasure law for assignment to show that f is refined by you answer to part (a).

Solution: We need to show that the erasure of $x > 0 \Rightarrow 0 \le x' < x[x' : x/2]$ is universally true. After making the substitution we have

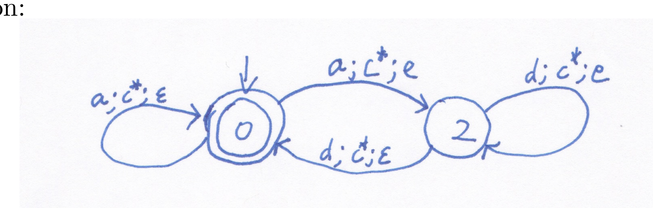
$$x > 0 \Rightarrow 0 \le x/2 < x$$

And there is nothing to erase. This is clearly true for all x.

Q4[6] In the following REFR, what do you get if state 1 is eliminated?

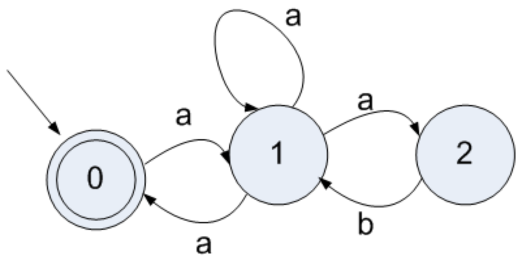


Solution:



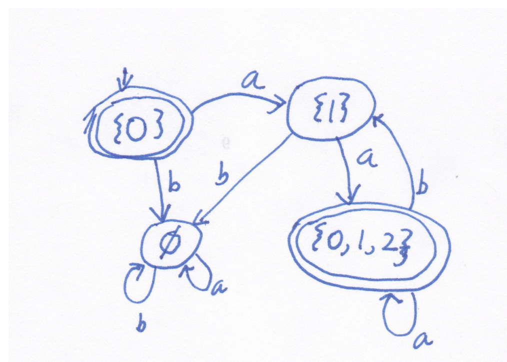
Note: I really should have first ensured the start state had no incoming transitions and that the final state had no outgoing transitions.

Q5 [8] Use subset construction to derive a DFR for following NDFR. (Assume the alphabet is {'a', 'b'}.)



Solution:

W	$\dot{q}$	a	$\dot{r}$
$\{\{0\}\}$	$\{0\}$	a	$\{1\}$
		b	$\emptyset$
$\{\{1\}, \emptyset\}$	$\{1\}$	a	$\{0,1,2\}$
		b	$\emptyset$
$\{\{0,1,2\}, \emptyset\}$	$\{0,1,2\}$	a	$\{0,1,2\}$
		b	$\{1\}$
$\{\emptyset\}$	$\emptyset$	a	$\emptyset$
	$\emptyset$	b	$\emptyset$
$\emptyset$			



Q6 [8] Use Thompson's construction to derive a NDFR for ('a' | 'b'; 'c')*

Solution: