

An Application

I have an if-statement

if(! (a < c || a != b) || a < c) { S } else { T }

How can I simplify this compound sentence?

Devote a variable to the primitive sentences

$$P : a < c$$

$$Q : a == b$$

Now we need to simplify: $\neg(P \vee \neg Q) \vee P$

Algebraic method

$$\begin{aligned} & \neg(P \vee \neg Q) \vee P \\ \Leftrightarrow & \underbrace{(\neg P \wedge \neg\neg Q)} \vee P && \text{De Morgan' law} \\ \Leftrightarrow & (\neg P \wedge Q) \vee P && \text{Involution} \\ \Leftrightarrow & \underbrace{(\neg P \vee P)} \wedge (Q \vee P) && \text{Distribute OR over AND} \\ \Leftrightarrow & T \wedge (Q \vee P) && \text{Excluded middle} \\ \Leftrightarrow & Q \vee P && \text{Identity} \end{aligned}$$

The simplified statement is

if(a == b || a < c) { S } else { T }

The Truth-Table method.

There are only 4 different truth assignments for P and Q .

We can use a table to calculate the value of each compound sentence in $\neg(P \vee \neg Q) \vee P$ in each case

P	Q	$\neg Q$	$(P \vee \neg Q)$	$\neg(P \vee \neg Q)$	$\neg(P \vee \neg Q) \vee P$
F	F	T	T	F	F
F	T	F	F	T	T
T	F	T	T	F	T
T	T	F	T	F	T

The final column matches the truth table for $P \vee Q$.