Applying ideas in graph theory

- Can you modify the adjacency matrix method of counting paths to calculate the shortest path instead? (Hint: try modifying the definition of matrix multiplication.)
- Suppose 2 (or more) robots are in an environment that consists of a number of 'places'.
  - ∗ At each time step each robot can stay where it is or can move to an adjacent place.
  - ∗ For safety, two robots can not be at the same place, nor can they be at adjacent places.
  - ∗ Given initial locations for each robot and destinations for each robot,
  - ∗ how we find the fastest safe plan for moving the robots to their destinations?
- Suppose a system has n states: $\{0, 1, ..., n-1\}$
  - ∗ On receiving a 1, in state i it moves to another state j with a certain probability p_{ij}
  - ∗ On receiving a 0, in state i it moves to another state

j with a certain probability q_{ij}

$$\forall i, \left(1 = \sum_j p_{ij} \wedge 1 = \sum_j q_{ij}\right)$$

  - ∗ Assume the system starts in state 0 and receives a string of 0s and 1s, say 100110.
  - ∗ How can we calculate the probabilities that the system is in each state?
- A sequence of tasks must be accomplished by a number of robots
  - ∗ Each task requires one or more robots in various roles.
  - ∗ In some cases the robot that plays a given role in one task must play a given role in a subsequent task (e.g. a robot that picks up a screwdriver in one task, might have to be the robot that uses the screwdriver in a subsequent task).
  - ∗ For a finite set of roles R and a finite set of tasks T with each task requiring some subset of roles. How can we assign robots to roles so that no robot has two roles in the same task?

- A graph has bounded degree k if every vertex has degree k or less. Does a simple graph of bounded degree k necessarily have a $(k+1)$-colouring?