Midterm

Engineering 3422, 2004

Friday, October 22

Q −1. What is your name? ________________

Q0[6]. In this question all variables represent integers.

"True necessarily", "false necessarily", or "depends on the integers", in each case.

- If $a \equiv b \pmod{m}$ and $a \equiv b \pmod{n}$ then $a \equiv b \pmod{mn}$________________
- If $m \mid a$ and $m \mid b$ then $m \mid ab$________________
- If $10 \mid a$ and $11 \mid a$ and $|a| < 100$.________________

Q1[6]. In this question, variables P, Q, and R are boolean, while S and T are sets. A and B are predicates on values in S.

Classify each of the following sentences as "tautology", "contradiction", "conditional sentence".

- $P \wedge (P \rightarrow Q) \leftrightarrow P \wedge Q$ ________________
- $S \cup (T - S) = S \cup T$________________
- $P \wedge (Q \leftrightarrow \neg P) \wedge Q$________________
- $\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, x > y$________________
- $\forall y \in \mathbb{Z}, \exists x \in \mathbb{Z}, x > y$________________
- $(\exists x \in S, A(x)) \wedge (\exists x \in S, B(x)) \rightarrow (\exists x \in S, A(x) \wedge B(x))$________________

Q2[4] Express using quatifier notation and the divisibility relation. S and T are sets of integers .

- Every integer in S divides some integer in T.
- No integer in S divides any integer in T.
- A unique integer in S divides every integer in T.

Q3[10]. Directly from the definitions of congruence and divisibility, show that, for all integers a and b, if $a \equiv 2 \pmod 5$ and $b \equiv 3 \pmod 5$ then $ab \equiv 1 \pmod 5$

Q4[4]. Simplify as much as possible

- $\{x \in \mathbb{N} \mid \exists m \in \mathbb{N}, x = 7 - 2m\} =$________________
- $(\forall a \in \mathbb{N}, \forall b \in \mathbb{N}, \exists x \in \mathbb{N}, \exists m \in \mathbb{N}, x = a - bm) \Leftrightarrow$________________

Q5[10]. Show that for all sets A and B, $A \cap B = A \cap \overline{B}$ implies that $A = \emptyset$.
