Q0. [6] True or false in each case. For propositional expressions A, B, and C and variable V:

- $A \leftrightarrow B$ is a contradiction if and only if A and B are not equivalent.*false* *If $A \leftrightarrow B$ is a conditional sentence then the expressions A and B will not be equivalent.*
- If $A \leftrightarrow B$ is a tautology then $A[V := C] \Leftrightarrow B[V := C]$.*true* *By the principle of replacement.*
- If $A \Leftrightarrow B$ then $C[V := A] \Leftrightarrow C[V := B]$.*true* *This is just another way of stating the principle of substitution.*

Q1. [8] Classify each of the following sentences as a "tautology", "contradiction", or a "conditional sentence". No proof is required

- $(P \vee Q) \wedge \neg P$ is a *conditional sentence* *It simplifies to $Q \wedge \neg P$*
- $P \wedge (P \rightarrow \neg P)$ is a *contradiction* *It simplifies to $P \wedge \neg P$*
- $P \leftrightarrow \neg P$ is a *contradiction*
- $(P \rightarrow Q) \wedge (Q \rightarrow P)$ is a *conditional sentence It simplifies to P*

Q2. [10] Give an algebraic proof of the following laws. Give a hint (the name of the law applied) for each step and underline to indicate the location of each use of the principle of substitution.

(a)

$$(P \to Q) \vee (R \to Q) \Leftrightarrow P \wedge R \to Q$$

Solution:

$$\begin{array}{ll} & \big(\underline{P \to Q}\big) \vee (\underline{R \to Q}) \\ \Leftrightarrow & (\neg P \vee Q) \vee (\neg R \vee Q) \ \text{ Definition of } \to \\ \Leftrightarrow & \underline{\neg P \vee \neg R} \vee Q \ \text{ Commutativity \& idempotence} \\ \Leftrightarrow & \neg(P \wedge R) \vee Q \ \text{ De Morgan} \\ \Leftrightarrow & P \wedge R \to Q \ \text{ Definition of } \to \end{array}$$

(b)

$$(\neg P \vee Q) \wedge (P \vee \neg Q) \Leftrightarrow P \leftrightarrow Q$$

Solution:

$$\begin{array}{ll} & \big(\underline{\neg P \vee Q}\big) \wedge \big(\underline{P \vee \neg Q}\big) \\ \Leftrightarrow & (P \to Q) \wedge (Q \to P) \ \text{ Definition of } \to \\ \Leftrightarrow & P \leftrightarrow Q \ \text{ Anti-symmetry of } \to \end{array}$$
