Quiz 1

Engineering 3422, 2004

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Q0. "True necessarily", "false necessarily", or "depends on the sets", in each case.

- If $A \cap B = \emptyset$ then $|A| + |B| = |A \cup B|$
  - *Necessarily true. For finite sets, it is easy to see this will be true.*
- If $A \subseteq B$ then $A \cup B = A$ and $A \cap B = B$
  - *Depends on the sets. If $A = B$ then it is true. If $A \subset B$ it is false.*
- $\emptyset \in A$
  - *Depends on the set. If $A = \emptyset$ it is false. If $A = \{\emptyset\}$ it is true.*
- If $A \subseteq B$ then $\mathcal{P}(A) \subseteq \mathcal{P}(B)$
  - *Necessarily true. Suppose $C \in \mathcal{P}(A)$. Then $C \subseteq A$ and hence $C \subseteq B$. So $C \in \mathcal{P}(B)$.*
- If $A \times B = \emptyset$ then either $A = \emptyset$ or $B = \emptyset$
  - *Necessarily true. If $a \in A$ and $b \in B$, then $(a, b) \in A \times B$.*
- If $\forall x \in A, x \in B$ then $\exists x \in A, x \in B$
  - *Depends on the sets. Consider $A = \emptyset$.*

Q1. Simplify as much as possible

- $\{x \in \mathbb{N} \mid x < 2\} = \{0, 1\}$
- $\{x \in \mathbb{N} \mid x < 0\} = \emptyset$
- $\{x \in \{0, 1, ..., 9\} \mid \exists y \in \mathbb{N}, y^2 = x\} = \{0, 1, 4, 9\}$
- $\neg\exists x \in A, (x \in \overline{B}) \Leftrightarrow A \subseteq B$ (Also acceptable: $\forall x \in A, (x \in B)$.)

Q2 Use quantifier notation to express the following English statements in predicate logic. A is an **array** of size $N \geq 0$ integers and $I = \{0, 1, ...N - 1\}$.

- Every number in I appears as as item of array A at least once.

$$\forall i \in I, \exists j \in I, (A[j] = i)$$

- No item of A is negative.

$$\neg\exists i \in I, (A[i] < 0)$$

or

$$\forall i \in I, A[i] \geq 0$$

- The items of array A are sorted in ascending order.

$$\forall i \in \{0, 1, ..., N - 2\}, (A[i] \leq A[i + 1])$$

or

$$\forall i \in I, \forall j \in I, (i \leq j \rightarrow A[i] \leq A[j])$$

Q3

Prove the following tautology for arbitrary sets A and B. Give a hint for each step. Use underlining to show where the principle of substitution is used.

$$(A - B) \cap C = (A - B) \cap (C - B)$$

$$
\begin{aligned}
& \underline{(A - B)} \cap C \\
= \quad & (A \cap \underline{\overline{B}}) \cap C \text{ Definition of } - \\
= \quad & A \cap \overline{B} \cap \overline{B} \cap C \text{ Idempotence of } \cap \\
= \quad & \underline{\big(A \cap \overline{B}\big)} \cap \underline{(C \cap \overline{B})} \text{ Commutativity and Associativity} \\
= \quad & (A - B) \cap (C - B) \text{ Definition of } -
\end{aligned}
$$