Quiz 1

Engineering 3422, 2004

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Q0. "True necessarily", "false necessarily", or "depends on the sets", in each case.

- If $A \cap B = \emptyset$ then $|A| + |B| = |A \cup B|$ _____
- If $A \subseteq B$ then $A \cup B = A$ and $A \cap B = B$ _____
- $\emptyset \in A$ _____
- If $A \subseteq B$ then $\mathcal{P}(A) \subseteq \mathcal{P}(B)$ _____
- If $A \times B = \emptyset$ then either $A = \emptyset$ or $B = \emptyset$ _____
- If $\forall x \in A, x \in B$ then $\exists x \in A, x \in B$ _____

Q1. Simplify as much as possible

- $\{x \in \mathbb{N} \mid x < 2\} =$
- $\{x \in \mathbb{N} \mid x < 0\} =$
- $\{x \in \{0, 1, ..., 9\} \mid \exists y \in \mathbb{N}, y^2 = x\} =$
- $\neg\exists x \in A, (x \in \overline{B}) \Leftrightarrow$

Q2 Use quantifier notation to express the following English statements in predicate logic. A is an **array** of size $N \geq 0$ integers and $I = \{0, 1, ...N-1\}$.

- Every number in I appears as as item of array A at least once.

- No item of A is negative.

- The items of array A are sorted in ascending order.

Q3

Prove the following tautology for arbitrary sets A and B. Give a hint for each step. Use underlining to show where the principle of substitution is used.

$$(A - B) \cap C = (A - B) \cap (C - B)$$
