Quiz 2

Engineering 3422, 2004

Wednesday Nov 10, 2004

Name _____

Q0 [12]

Consider a one-way infinite sequence defined as follows

$a(0) = 0$

$a(n) = 4 \times a(n/2)$, if $n \geq 2$ and n is even

$a(n) = 2 \times a(n-1) + 2 \times n - 1$, if $n \geq 1$ and n is odd

Use *the principle of complete induction* to prove that $\forall n \in \mathbb{N}$, $a(n) = n^2$.

(a)[2] What must be proved in the base step (only one base step is needed)

(b)[2] Prove the base step.

(c)[4] What must be proved in the inductive step? Be clear about exactly how variables are quantified.

(d)[4] Prove the inductive step, being careful to clearly state the induction hypothesis and to clearly indicate where the induction hypothesis is being used.

Q1 [9]

Suppose you want to prove that any finite set S, such that $|S| \geq 3$, has $|S|\,(|S|-1)(|S|-2)/6$ subsets of size 3, using simple induction.

(a)[3] State the property that must be proved for all $n \in \{3, 4, ...\}$. Be careful to properly quantify all variables.

$P(n)$ is:

(b)[3] Without reference to P, state what must be proved in the base step. Be clear about exactly how variables are quantified.

(c)[3] Without reference to P, state what must be proved in the inductive step. Be clear about exactly how variables are quantified.
