Quiz 3 - Solution

Engineering 3422, 2004

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Name: Solution.
[10] Q0. Find a closed form solution for the sequence defined by:

$$
\begin{aligned}
a(0) &= 3 \\
a(1) &= 1 \\
a(n) &= -a(n-1) + 6 \cdot a(n-2)
\end{aligned}
$$

Solution: The characteristic equation of the recurrence relation is: $x^2 + x - 6 = 0$. This has roots $r_1 = -3$ and $r_2 = 2$. The general solution of the recurrence relation is thus $a(n) = \theta_1(-3)^n + \theta_2 2^n$. From the first two terms of the recurrence, we get the following system of equations for the θs:

$$
\begin{aligned}
\theta_1 + \theta_2 &= 3 \\
-3\theta_1 + 2\theta_2 &= 1
\end{aligned}
$$

From the first equation, $\theta_2 = 3 - \theta_1$. Substituting in the second equation yields $6 - 5\theta_1 = 1 \Rightarrow \theta_1 = 1$. So $\theta_2 = 3 - 1 = 2$ and $a(n) = (-3)^n + 2^{n+1}$.

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[10] **Q1.** Suppose that $\mathrm{dom}(S) = \mathrm{rng}(R)$. Show that if S and R are both total relations, then so is $S \circ R$.

Solution: Let $X = \mathrm{dom}(R), Y = \mathrm{dom}(S) = \mathrm{rng}(R)$, and $Z = \mathrm{rng}(S)$.

- Let x be any member of X.
- By the definition of total relation, it remains to show there is a z such that $x(S \circ R)z$
- By the definition of total relation and since R is total, $\exists y \in Y, xRy$.
- Let y be such a value. Note that $y \in \mathrm{rng}(R) = \mathrm{dom}(S)$
- By the definition of total relation and since S is total, $\exists z \in Z, ySz$.
- Let z be such a value.
- Since xRy and ySz, $x(S \circ R)z$

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[5] **Q2.** Let S and R be relations with domain and range both equal to $\{0, 1, 2, 3, 4, 5\}$. Define the graphs by

$$xRy \text{ iff } y = x \bmod 3$$
$$xSy \text{ iff } y = (x + 1) \bmod 6$$

List all the members of the graph of $S \circ R$.

Solution: $\mathrm{graph}(S \circ R) = \{(0, 1), (1, 2), (2, 3), (3, 1), (4, 2), (5, 3)\}$.
