An Application

I have an if-statement
if(! (a<c || a != b) || a<c) { S } else { T }

How can I simplify this compound sentence?

Devote a variable to the primitive sentences

$P:\ a < c$
$Q:\ a == b$

Now we need to simplify: $\neg(P \vee \neg Q) \vee P$

Algebraic method

$$
\begin{aligned}
& \overline{\neg(P \vee \neg Q)} \vee P \\
\Leftrightarrow\ & (\neg P \wedge \overline{\neg\neg Q}) \vee P && \text{De Morgan' law} \\
\Leftrightarrow\ & (\neg P \wedge Q) \vee P && \text{Involution} \\
\Leftrightarrow\ & \overline{(\neg P \vee P)} \wedge (Q \vee P) && \text{Distribute OR over AND} \\
\Leftrightarrow\ & T \wedge (Q \vee P) && \text{Excluded middle} \\
\Leftrightarrow\ & Q \vee P && \text{Identity}
\end{aligned}
$$

The simplified statement is
if(a==b || a < c) { S } else { T }

The Truth-Table method.

There are only 4 different truth assignments for P and Q.

We can use a table to calculate the value of each compound sentence in $\neg(P \vee \neg Q) \vee P$ in each case

P	Q	$\neg Q$	$(P \vee \neg Q)$	$\neg(P \vee \neg Q)$	$\neg(P \vee \neg Q) \vee P$
F	F	T	T	F	F
F	T	F	F	T	T
T	F	T	T	F	T
T	T	F	T	F	T

The final column matches the truth table for $P \vee Q$.