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An Application

I have an if-statement if(! ($a < c \parallel a \parallel = b$) || a < c) { *S* } else { *T* } How can I simplify this compound sentence? Devote a variable to the primitive sentences

$$P: a < c$$
$$Q: a == b$$

Now we need to simplify: $\neg (P \lor \neg Q) \lor P$

Algebraic method

 $\begin{array}{l} \begin{array}{l} \begin{array}{l} \neg (P \lor \neg Q) \lor P \\ \Leftrightarrow \overline{(\neg P \land \neg \neg Q)} \lor P & \text{De Morgan' law} \\ \Leftrightarrow \overline{(\neg P \land Q)} \lor P & \text{Involution} \\ \Leftrightarrow \overline{(\neg P \lor P)} \land (Q \lor P) & \text{Distribute OR over AND} \\ \Leftrightarrow \overline{T \land (Q \lor P)} & \text{Excluded middle} \\ \Leftrightarrow Q \lor P & \text{Identity} \end{array} \\ \begin{array}{l} \text{The simplified statement is} \\ \text{if(a==b || a < c) { S } else { T } \end{array}} \end{array}$

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The Truth-Table method.

There are only 4 different truth assignments for P and Q. We can use a table to calculate the value of each

compound sentence in $\neg(P \lor \neg Q) \lor P$ in each case

				$\neg (P \vee \neg Q)$	$\neg (P \vee \neg Q) \vee P$
F	F	Т	Т	F	F
F	T	F	F	Т	Т
T	F	T	Т	F	Т
T	T	F	Т	F	Т

The final column matches the truth table for $P \lor Q$.

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