

# An Application

I have an if-statement

if( ! ( a < c || a != b ) || a < c ) { S } else { T }

How can I simplify this compound sentence?

Devote a variable to the primitive sentences

$$P : a < c$$

$$Q : a == b$$

Now we need to simplify:  $\neg(P \vee \neg Q) \vee P$

## Algebraic method

$$\begin{aligned} & \neg(P \vee \neg Q) \vee P \\ \Leftrightarrow & \underline{\neg(P \vee \neg Q)} \vee P && \text{De Morgan' law} \\ \Leftrightarrow & (\neg P \wedge \underline{\neg\neg Q}) \vee P && \text{Involution} \\ \Leftrightarrow & (\neg P \wedge Q) \vee P && \text{Distribute OR over AND} \\ \Leftrightarrow & \underline{(\neg P \vee P)} \wedge (Q \vee P) && \text{Excluded middle} \\ \Leftrightarrow & T \wedge (Q \vee P) && \text{Identity} \\ \Leftrightarrow & Q \vee P \end{aligned}$$

The simplified statement is

if( a == b || a < c ) { S } else { T }

## The Truth-Table method.

There are only 4 different truth assignments for  $P$  and  $Q$ .

We can use a table to calculate the value of each compound sentence in  $\neg(P \vee \neg Q) \vee P$  in each case

$P$	$Q$	$\neg Q$	$(P \vee \neg Q)$	$\neg(P \vee \neg Q)$	$\neg(P \vee \neg Q) \vee P$
$F$	$F$	$T$	$T$	$F$	$F$
$F$	$T$	$F$	$F$	$T$	$T$
$T$	$F$	$T$	$T$	$F$	$T$
$T$	$T$	$F$	$T$	$F$	$T$

The final column matches the truth table for  $P \vee Q$ .