

Suppose that A and B are matrices that commute. Show that $AB^n = B^n A$.

As you may know, you can determine the remainder of a number divided by 9 by adding up its digits, repeating as needed until only one digit remains.[1] Show that, for all natural n and natural sequences q

$$\sum_{i=0}^{n} 10^i q_i \equiv \sum_{i=0}^{n} q_i \pmod 9$$

Use induction on n.

Show the existence part of the fundamental theorem of arithmetic using complete induction.

That is, for all integers n greater than 1 there is a sequence of primes that multiplied together gives n.

Consider the following game. There is a pile of $m > 0$ marbles. Each player takes turns removing 1, 2, or 3 marbles. The player who *does not* take the last marble is the winner. Show that the first player can force a win if the number of marbles is originally $4j$, $4j + 2$, or $4j + 3$, for some $j \in \mathbb{N}$. Show that the second player can force a

[1] Of course, if the final digit is 9, the remainder is actually 0.

win if the number of marbles is originally $4j+1$, for some $j \in \mathbb{N}$.

Show that $3 \mid n^3 + 2n$ for all $n \in \mathbb{N}$

A cyclic grey-code of width m is a list of binary sequences, each of width m bits, such that any two adjacent sequence differ at exactly one position and furthermore the first and the last differ at exactly one position, and all the sequences differ E.g.

$$\langle 0,0,0\rangle, \langle 0,0,1\rangle, \langle 0,1,1\rangle, \langle 0,1,0\rangle,$$
$$\langle 1,1,0\rangle, \langle 1,1,1\rangle, \langle 1,0,1\rangle, \langle 1,0,0\rangle$$

is a cyclic grey-code of width 3 and length 8. Show that for all $m \in \mathbb{N}$, there is a cyclic grey-code of width m and length 2^m.