Problem Set 0

Engineering 3422, 2005

To be done for Friday Sept 16th

Q0

(a) Think of another application of graph colouring.

(b) Suppose that we can show that there is no "fast"[1] algorithm to colour graphs in general. Does it follow that there is no fast algorithm for the exam scheduling problem?

(c) It is known that if you can draw a graph so that no two edges (lines) cross, then it can be coloured with four or fewer colours.[2] Consider testing single-layer PCB boards.[3] What does this result tell us about that? What does it tell us about the other problems?

Q1

(a) In the Java programming language a rule states that

> "fields that are both static and final, or are declared in an interface, must have an initializer".

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[1] Exactly what "fast" means in terms of algorithms is something you will see in term 4. For this problem it suffices to know that we can converting any graph to a classlist is considered "fast" and that an algorithm that consists of two "fast parts" done one after the other is considered fast".

In fact, for the most common definition of "fast" (something called polynomial-time) no one yet knows whether there is a fast graph colouring algorithm. The question is equivalent to one of the great unsolved problems in math: The $P = NP$ problem.

[2] You are allowed to draw the nodes anywhere in the plane and to draw the edges as arbitrary curves. Such a graph is called "planar".

This result, known as the "four colour theorem", was proved in 1976 by Haken and Appel, about 90 years after the problem was first posed. The long awaited proof created some controversy as it was partially computer assisted and included a few thousand pages of computer print-outs. To really check the proof one either had to check all the print-outs, or check the correctness of a nontrivial computer program.

[3] A single layer PCB board only has one metallic layer.

Assign a propositional variable to each of the relevant primitive statements and write a propositional expression that expresses this rule symbolically as succinctly as you can.

(b) Also in Java, there is a rule that states

> "a method must have a body, if the method is not declared in an interface and is not declared abstract"

and another that states

> "a method declared abstract or in an interface must not have a body".

Assign a propositional variable to each of the relevant primitive statements and write a propositional expression that expresses both these rules symbolically as directly as possible.

(c) Use algebraic simplification, if necessary, to simplify your answer to use a minimum of operations. For each step, remember to use the name of the law applied as a hint.

Q2 In English, write all the derived implications for the following: If you get an A in this course, then you pass the course. Label each derived implication with its kind (contrapositive, inverse, converse)

Q3 Use the table method to determine if each of the following propositional expressions is a tautology, a contradiction, or a conditional statement.

(a) $(P \rightarrow Q) \wedge P \wedge (\neg P \vee Q)$

(b) $P \vee Q \vee \neg P \vee R$

(c) $P \wedge (Q \leftrightarrow P) \wedge R$

Q4 Use truth tables to verify the following laws

$$\begin{aligned} \text{Contrapositive:} \quad P \rightarrow Q \quad &\Leftrightarrow \quad \neg Q \rightarrow \neg P \\ \text{Shunting:} \quad P \wedge Q \rightarrow R \quad &\Leftrightarrow \quad P \rightarrow (Q \rightarrow R) \end{aligned}$$

Q5 Two Canadian coins add up to 30¢, yet one of them is not a nickel. Explain how this can be.